Accuracy of the hypothetical sky-polarimetric Viking navigation versus sky conditions: revealing solar elevations and cloudinesses favourable for this navigation method

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According to Thorkild Ramskou’s theory proposed in 1967, under overcast and foggy skies, Viking seafarers might have used skylight polarization analysed with special crystals called sunstones to determine the position of the invisible Sun. After finding the occluded Sun with sunstones, its elevation angle had to be measured and its shadow had to be projected onto the horizontal surface of a sun compass. According to Ramskou’s theory, these sunstones might have been birefringent calcite or dichroic cordierite or tourmaline crystals working as polarizers. It has frequently been claimed that this method might have been suitable for navigation...
even in cloudy weather. This hypothesis has been accepted and frequently cited for decades without any experimental support. In this work, we determined the accuracy of this hypothetical sky-polarimetric Viking navigation for 1080 different sky situations characterized by solar elevation $\theta$ and cloudiness $\rho$, the sky polarization patterns of which were measured by full-sky imaging polarimetry. We used the earlier measured uncertainty functions of the navigation steps 1, 2 and 3 for calcite, cordierite and tourmaline sunstone crystals, respectively, and the newly measured uncertainty function of step 4 presented here. As a result, we revealed the meteorological conditions under which Vikings could have used this hypothetical navigation method. We determined the solar elevations at which the navigation uncertainties are minimal at summer solstice and spring equinox for all three sunstone types. On average, calcite sunstone ensures a more accurate sky-polarimetric navigation than tourmaline and cordierite. However, in some special cases (generally at $35^\circ \leq \theta \leq 40^\circ$, 1 okta $\leq \rho \leq 6$ oktas for summer solstice, and at $20^\circ \leq \theta \leq 25^\circ$, 0 okta $\leq \rho \leq 4$ oktas for spring equinox), the use of tourmaline and cordierite results in smaller navigation uncertainties than that of calcite. Generally, under clear or less cloudy skies, the sky-polarimetric navigation is more accurate, but at low solar elevations its accuracy remains relatively large even at high cloudiness. For a given $\rho$, the absolute value of averaged peak North uncertainties dramatically decreases with increasing $\theta$ until the sign $(\pm)$ change of these uncertainties. For a given $\theta$, this absolute value can either decrease or increase with increasing $\rho$. The most advantageous sky situations for this navigation method are at summer solstice when the solar elevation and cloudiness are $35^\circ \leq \theta \leq 40^\circ$ and 2 oktas $\leq \rho \leq 3$ oktas.

1. Introduction

Using easily recognizable coastal places and other simple aids [1–3], the Vikings discovered new areas between the ninth and thirteenth centuries like Iceland, Greenland and the coast of North America. In the North Atlantic region, they established colonies that were connected to the European continent through permanent trading routes when the seawater was free of ice [4–6]. They regularly covered long distances on the ocean lasting several weeks without any modern navigation equipment, such as magnetic compass (e.g. [7]). Solver [8] described a navigation method with which the Viking seafarers could have oriented themselves on the open ocean in cloudless weather. His theory is supported by an archeological artefact discovered in 1948, when a fragment of a wooden dial was found under the ruins of a Benedictine convent near the Uunartoq fjord in Greenland [9]. According to the most possible explanations, the dial might have been a fragment of a sun compass, a navigation tool with which the Viking navigators could determine the geographical North with the help of the shadow of a vertical gnomon cast by the Sun [2,8–12]. There are, however, alternative explanations for the usage of this device proposed by Bernáth et al. [13,14].

According to the theory of Ramskou [15] under totally overcast or foggy sky, the Vikings might have used skylight polarization analysed with special tools called sunstones to determine the position of the invisible Sun. After finding the occluded Sun with sunstones, its elevation angle had to be measured and its shadow had to be projected onto the horizontal surface of a sun compass. It has been frequently claimed that this method might also have been suitable for navigation in cloudy weather, even in overcast [16–18]. Reference to sunstones can be found in ancient Viking legends, the sagas, being described as tools enabling the determination of the Sun’s position behind clouds [19–21]. According to Ramskou’s theory, these sunstones might have been birefringent calcite or dichroic cordierite, tourmaline or andalusite minerals (as supposed also by many other researchers, [16–18,22–28]) that work as polarizers, that is the observer can perceive radiance changes in the skylight coming through. Viking navigators also might have been able to determine the position of the occluded sun with the Haidinger’s brushes instead of sunstones [26,29,30].
Ramskou’s hypothesis has been accepted and frequently cited for decades without measuring its accuracy under different circumstances. Using imaging polarimetry, we have measured the atmospheric optical prerequisites of the hypothetical sky-polarimetric Viking navigation [31–37]. The steps of this navigation method are the following:

— **First step:** After calibration of the sunstones in cloudless weather by marking the direction pointing towards the Sun in a well-recognizable crystal alignment (e.g. where the radiance of transmitted skylight is minimal or maximal), the Viking navigator might have adjusted the sunstones in cloudy weather at two different celestial points by rotating both in front of his eyes, through which he could determine the directions perpendicular to the local direction of skylight polarization.

— **Second step:** At this sunstone alignment, the previously carved markings gave the directions of two celestial great circles, in the intersection of which the occluded Sun could be found. The navigator had to determine this intersection by naked eye.

— **Third step:** Once the position of the Sun was found, the navigator had to measure its vertical elevation with his fists and fingers, in order to reproduce the Sun shadow to use the sun compass.

— **Fourth step:** Finally, the shadow-stick of the sun compass should be aligned parallel to the meridian (vertical azimuth plane) of the invisible Sun.

All four steps have specific uncertainty functions, contributing separately to the North uncertainty $\Delta \omega_N$, the degree with which the estimated northern direction differs from the real geographical North. These uncertainties have been measured in psychophysical laboratory/planetarium experiments, and the accuracy of navigation was calculated assuming that one of these steps was erroneous and the other three steps were accurate (having no uncertainty): (i) Száz et al. [38] found that in the first step, the sunstone adjustment is more accurate if dichroic tourmaline and cordierite are used when the degree of polarization $p$ of skylight is higher than a critical value $p^*$ (20% for cordierite and 40% for tourmaline), while for $p < p^*$ a calcite sunstone can be more accurately adjusted. However, the accuracy of calcite adjustment greatly depends on the crystal quality, because superficial or inner scratches and contamination can disturb or deceive the navigator. (ii) Farkas et al. [39] measured the accuracy of intersection estimation of the two great circles in the second step and found that at lower solar elevations ($5^\circ < \theta < 25^\circ$), the test persons measured the antisolar point instead of the Sun in some cases, resulting in high navigation uncertainty. In the calculation of uncertainty propagation, we removed these measurements of the antisolar points coming from the second step, and thus, they did not distort our results. Furthermore, such sun versus antisun misestimations might have not been a severe problem in real life, because the antisolar point is below the horizon during daytime, and if such a spurious mismeasurement occurred (only very sporadically), then the Viking navigators would have likely ignored such wildly discrepant subhorizon solar position estimates using their time sense and the sky brightness (implying also that in the given point of time, the sun cannot be under the horizon). Only at sunset and sunrise can the antisun be deceiving, when it is exactly on the horizon, then the Vikings could use their knowledge on the earlier sailing direction (if a navigator used the misestimated antisun instead of the correct sun, the new sailing direction would turn by about 180° relative to the earlier one, and such a too large turn would indicate that a misestimation of the solar position happened). The North uncertainty is also high if the two reference points are far from each other. Thus, the navigation is more accurate around the summer solstice, when the solar elevation is the highest. (iii) Száz et al. [40] measured the accuracy of elevation estimation in the third step. Although both the elevation uncertainty and the North uncertainty increased with solar elevation, 48% of all elevation estimations were more accurate than $\pm 1^\circ$. (iv) In this work, we measured the uncertainty function of the fourth step in a planetarium.

Using the uncertainty functions of the four steps of sky-polarimetric navigation ([38–40]; this study), in this synthesis work, we determined the navigation (North) uncertainties under
1080 different meteorological conditions, the sky polarization patterns of which were measured by full-sky imaging polarimetry. These 1080 skies differed in the solar elevation \( \theta \) and the cloud per cent \( \rho \), and were selected from the 1296 different meteorological conditions used by Horváth et al. [30] by omitting the subhorizon cases. The knowledge on sky polarization is reviewed in earlier studies [20,21,31–33,37,41–44]. The aim of this work is to determine and analyse the accuracy of sky-polarimetric navigation under these many different sky conditions and to answer the most important question of this topic: under what meteorological conditions this hypothetical navigation method could be used? We would like to emphasize that with our study, we do not state that we applied the same method as the Vikings used (because nobody knows how the Vikings have really navigated under cloudy and foggy conditions), and we are not trying to prove that the Vikings used this sky-polarimetric navigation. Our aim was to reveal the values of solar elevation and sky cloudiness favourable for this navigation method.

2. Material and methods

(a) Measuring the uncertainty function of the fourth step of sky-polarimetric navigation

In the planetarium of the Eötvös University, we measured the uncertainty function of the fourth step of sky-polarimetric navigation. A black dot with angular extension of 0.25° was projected on the white planetarium dome with an azimuth angle \( \phi \) (ranging between \(-45°\) and \(+45°\) from an arbitrary horizontal reference direction) and an elevation angle \( \theta \) (ranging from \(0°\) to \(55°\)). The test person sitting in the centre of the planetarium had to estimate the azimuth angle of this dot with the help of a digital goniometer modelling the Viking sun compass (electronic supplementary material, figure S1). The person had to rotate a shadow-stick-shaped elongated metal plate (7 cm long, tilted with \(45°\) from the horizontal) along its vertical axis until its long axis became parallel to the estimated azimuth direction of the projected dot. The difference \( \Delta = \phi_e - \phi_t \) between the estimated \( \phi_e \) and the true \( \phi_t \) azimuth angles of the dot was registered. The test person had to perform this estimation for 48 dots with random \( \phi \)- and \( \theta \)-values. We performed this measurement 10 times with 10 male test persons, whose ages ranged between 23 and 54 years. For a given \( \theta \), the resulting \( 10 \times 10 \times 48 = 4800 \) different \( \Delta \)-values were averaged, thus we obtained the average \( \langle \Delta \rangle \pm \) standard deviation (s.d.) \( \sigma_\phi \) for 48 different \( \theta \)-values. Since the average \( \langle \Delta \rangle \)-values approximated zero (as expected), the uncertainty function of the fourth step of sky-polarimetric navigation was defined as \( e_{\text{th}}(\theta) = \sigma_\phi(\theta) \). Finally, a parabola was fitted to the measured \( \sigma_\phi(\theta) \) values. Further on, this parabola is considered as the uncertainty function of the fourth step.

(b) Selection of 1080 different meteorological situations

The patterns of the degree of polarization \( p \) of skylight were measured by imaging polarimetry, the method of which is described in detail by Barta et al. [44]. Data on sky polarization have been collected with an automatic full-sky imaging polarimeter set-up in the Gothard Astronomical Observatory of the Eötvös University, Szombathely, Hungary (47°15’29.83’’ N, 16°36’15.67’’ E). In the last 3 years, this polarimeter functioned continuously and measured several tens of thousands of sky polarization patterns, from which we selected 1080 different skies. We grouped these skies on the basis of the following two parameters: (i) elevation angle \( \theta \) of the Sun above the horizon ranged from \(0°\) to \(50°\) (higher solar elevations did not occur at the \(61°\) northern latitude, the main Viking sailing route). This \( \theta \)-interval was divided into 10 equal intervals with an increment of \(5°\) as follows: \(0° \leq \theta_1 < 5°\), \(5° \leq \theta_2 < 10°\), \(10° \leq \theta_3 < 15°\), \(15° \leq \theta_4 < 20°\), \(20° \leq \theta_5 < 25°\), \(25° \leq \theta_6 < 30°\), \(30° \leq \theta_7 < 35°\), \(35° \leq \theta_8 < 40°\), \(40° \leq \theta_9 < 45°\), \(45° \leq \theta_{10} < 50°\). (ii) Cloud coverage \( \rho \) (per cent of the full sky covered by clouds) was determined with the use of the cloud detection algorithm \( k\text{NN} \) (\(k \) Nearest Neighbour) described by Barta et al. [44]. The interval \(0\% \leq \rho \leq 100\%\) was divided into nine categories, as in meteorology oktas are common units for estimating the cloud coverage of the visible sky region by eyesight [45]. Oktas from 0 to 8 refer to more and more intense cloud coverage based on the division of eight equal intervals.
when all four navigation steps have their own uncertainty function. This earlier algorithm was extended and made suitable to calculate the uncertainty propagation for computing the uncertainty propagation in the first step of sky-polarimetric navigation. with a custom-made computer programme. The basics of this algorithm were used by Száz we calculated the uncertainty propagation through the four steps of sky-polarimetric navigation in a different study.

Beside the cloud coverage $\rho$, another determinant could be the cloud thickness $t$. Since our cloud detection algorithm recognized clouds with almost all different $t$-values, cloud thickness is involved in determinant $\rho$. Thus, we did not consider $t$ as a separate variable.

Based on the above, we created $10 \times 9 = 90 \ (\theta, \rho)$ groups. In each group, we selected 12 different skies from our polarimetric sky archives. Finally, we obtained $90 \times 12 = 1080$ different sky situations differing in $\theta$ and $\rho$, but in a given group their $\theta$- and $\rho$-values were similar. Further on, we used the polarization patterns of these skies measured by imaging polarimetry in the green ($550 \text{ nm}$) spectral range, in which the human eye is most sensitive [47].

### (c) Uncertainty propagation and North uncertainty determination

We determined the navigation uncertainty, that is the North uncertainty $\Delta \omega_N$ for the 1080 different skies, supposing that Viking navigators used one of the three different sunstone crystals: calcite, cordierite or tourmaline. Since, in the literature, mainly these three crystal types have been assumed to be used as sunstones due to their predominant abundance compared to andalusite, the uncertainty function of the first step of sky-polarimetric navigation has been measured by Száz et al. [38] only for calcite, cordierite and tourmaline. To quantify the navigation uncertainty, we calculated the uncertainty propagation through the four steps of sky-polarimetric navigation with a custom-made computer programme. The basics of this algorithm were used by Száz et al. [38] for computing the uncertainty propagation in the first step of sky-polarimetric navigation. This earlier algorithm was extended and made suitable to calculate the uncertainty propagation when all four navigation steps have their own uncertainty function. $\Delta \omega_N$ was computed with the following algorithm:

- In a given sky, we excluded areas with $p < 5\%$ (being under the sensitivity threshold of the human eye) [38] and $p > 90\%$ (such high $p$-values were caused by the motion artefact of clouds and normally do not occur in real skies [20,21,43], and pixels of field objects not belonging to the sky).

- We chose point pairs $(m_1, m_2)$ from the non-excluded sky areas as follows: (i) the first point $m_1$ was chosen from a celestial quadratic grid with a side length of 30 pixels (figure 1a). (ii) The second point $m_2$ was chosen from a celestial polar grid with 20° resolution between angular distances $45^\circ \leq \tau \leq 90^\circ$ from $m_1$ (figure 1b). According to our earlier field experience with sunstones [14,36,39], point $m_2$ cannot be too close to $(0^\circ \leq \tau \leq 45^\circ)$ or too far $(90^\circ \leq \tau \leq 180^\circ)$ from $m_1$; otherwise, the accuracy of the second step of sky-polarimetric navigation decreases considerably.

- Using the measured degrees of polarization $p_1$ and $p_2$ in sky points $m_1$ and $m_2$, we calculated the uncertainties $e_1 = e(p_1)$ and $e_2 = e(p_2)$ of sunstone adjustment, where $e(p)$ is the uncertainty function of the first step measured in psychophysical laboratory experiments for cordierite, tourmaline and (best-performing) calcite crystals [38].

- Let $C_{1E}$ and $C_{2E}$ be the great circles passing through the sunstone centres $m_1$ and $m_2$ parallel to the straight markings engraved into the sunstone surface during calibration.
The estimated Sun position $E$ is the intersection of circles $C_{1E}$ and $C_{2E}$ (figure 1c). Let $C_{1S}$ and $C_{2S}$ be the celestial great circles connecting the Sun $S$ with points $m_1$ and $m_2$ (figure 1d). For each member $m_i$ of the point pair $m_1$ and $m_2$, we considered the two great circles $C_{i+}$ and $C_{i-}$ enclosing an angle of $2\epsilon_i(p_i)$ with each other around the great circle $C_S$ connecting points $m_i$ and $S$, where $i = 1, 2$. $C_{i+}$ and $C_{i-}$ enclose an angle of $+\epsilon_i(p_i)$ and $-\epsilon_i(p_i)$ with $C_S$, respectively (figure 1d). The intersections of circles $C_{i+}$, $C_{i-}$ and $C_{2+}$, $C_{2-}$ appoint a spherical tetragon (marked with grey in figure 1d) involving the real Sun position $S$. Owing to the maximum uncertainties $\pm \epsilon_i(p_i)$ of sunstone adjustments, all possible estimated Sun positions $E$ are within this grey tetragon for the given point pair $m_1$ and $m_2$. The area of this tetragon was divided into 100 points along a grid with tenth of the tetragon’s side length. Thus, we got 100 estimated Sun positions $E$ with which we computed further on. Such trimming of data points was necessary to reduce the computation time of the algorithm.

To determine the uncertainty of the second navigation step, we characterized the situations with the same four free parameters as Farkas et al. [39]: (i) the elevation angle $\theta_E$ of the estimated Sun, (ii) $\gamma_1$ and (iii) $\gamma_2$ meaning the angular distance of $m_1$ and $m_2$ from the estimated Sun $E$ and (iv) angle $\delta$ enclosed by the planes of the two celestial great circles mentioned. The used parameter intervals were based on the psychophysical planetary experiment of Farkas et al. [39]: $\theta_S$: 5°–25°, 35°–55°; $\gamma_1$: 35°–55°, 65°–85°, 95°–115°; $\delta$: 35°–55°, 65°–85°, 95°–115°, 125°–145°. Thus, we created from these intervals $2 \times 3 \times 3 \times 4 = 72$ parameter groups ($\theta_S$, $\gamma_1$, $\gamma_2$, $\delta$) and characterized them with the averaged elevation uncertainty $\Delta \theta$ and azimuth uncertainty $\Delta \phi$ that define a vector $(\Delta \theta, \Delta \phi)$ in the spherical coordinate system that mathematically describes the visually spherical sky dome. From the estimated Sun positions $E$, we selected those that could be sorted into one of the 72 parameter intervals, then these points were shifted with vector $(\Delta \theta, \Delta \phi)$ on the surface of the hypothetical sky dome (figure 1c) in order to add elevation and azimuth uncertainties originating from the second navigation step.

We determined the vertical elevation of the shifted estimated Sun positions $E_{sh}$ and calculated the elevation uncertainty $\Delta \theta_E$ based on the uncertainty function of the third step of sky-polarimetric navigation [40]. For each $E_{sh}$, we got an interval between $E_{sh} - \Delta \theta_E$ and $E_{sh} + \Delta \theta_E$ that was divided into equal vertical angular distances by 0.2°. Thus, instead of one $E_{sh}$ point, we got several points based on the elevation uncertainty of the third step, marked with $P_\theta$.

In the fourth step, the navigator had to align the shadow-stick of the sun compass parallel to the meridian (vertical azimuth plane) of the invisible Sun. The uncertainty of this alignment was characterized by the measured $\epsilon_{sh}(\theta)$ function, and for each Sun position $P_\theta$, we got an interval between $\phi_S(\theta) - \epsilon_{sh}(\theta)$ and $\phi_S(\theta) + \epsilon_{sh}(\theta)$ that was divided into equal horizontal angular distances by 0.2° perpendicular to the meridian, where $\phi_S(\theta)$ is the azimuth angle of point $P_\theta$. Thus, instead of each point $P_\theta$, we got several points based on the elevation uncertainty of the fourth step that gives the estimated Sun positions $P_E$ if the uncertainties of all the four steps are calculated in the estimation (figure 1g).

The Viking navigator derived the direction (angle) $\omega_N$ of the geographical North using the sun compass as follows (figure 1f): he might have determined the direction of the imaginary light rays originating from position $P_E$ of the invisible Sun with a shadow-stick [14,36]. After the horizontal Viking sun compass is rotated until the shadow tip of the vertical gnomon falls on the appropriate gnomonic line engraved in the compass’ surface, the symmetry axis of the gnomonic line would exactly point towards the geographical North, if there were no uncertainties. Since in reality the four navigation steps have more or less uncertainties, the symmetry axis of the gnomonic line points towards a direction that differs from the geographical North with an angle $\omega_N$ when the shadow tip falls on the gnomonic line. Since the gnomonic line is well visible, we considered the minimal uncertainty of this rotation (which did not affect the navigation) as negligible. This angle $\omega_N$ is the navigation uncertainty belonging to (i) a specific pair of sky points $m_1$ and $m_2$
with uncertainties $e_1$ and $e_2$ of sunstone adjustment in the first step, (ii) a parameter group $(\theta_S, \gamma_1, \gamma_2$ and $\delta$) with elevation and azimuth uncertainties ($\Delta \theta$, $\Delta \phi$) of intersection finding in the second step, (iii) an estimated solar elevation $\theta_E$ with elevation uncertainty $\Delta \theta_E$ in the third step and (iv) an estimated azimuth error of the shadow-stick alignment $e_4th(\theta)$ of sky-polarimetric navigation in the fourth step for a given date (e.g. spring equinox or summer solstice).

The navigation (or North) uncertainty $\Delta \omega_N$ for a given sky was calculated similarly as in Száz et al. [38]: angles $\omega_N$ for each estimated sun position $P_E$ were collected into a histogram, which were smoothed (convoluted) by a normalized Gaussian function with $5^\circ$ half-kernel size and $5^\circ$ standard deviation. This smoothed curve represented the distribution of the North uncertainty $\Delta \omega_N$ with a maximum at angle $\omega_{max}$ and a half bandwidth $\Delta \omega$ meaning the full width at half maximum (figure 1h). The smaller the $|\omega_{max}|$ and $\Delta \omega$, the more accurate the sky-polarimetric navigation.

There are always two possibilities to project the estimated sun position $P_E$ onto the gnomonic line (figure 1i): either (i) in the forenoon (when the sun compass is rotated until the shadow tip falls on the forenoon half of the gnomonic line) or (ii) in the afternoon (when the sun compass is rotated until the shadow tip falls on the afternoon half of the gnomonic line). Thus, we determined the navigation uncertainties $\Delta \omega_N$ for both forenoon and afternoon.

We performed the above calculations (uncertainty propagation) with the use of the uncertainty functions $e(p)$ measured for three different sunstone crystals (calcite, cordierite and tourmaline) for two astronomically momentous dates (spring equinox and summer solstice, figure 1j) and for forenoon and afternoon. This means $3 \times 2 \times 2 = 12$ $\Delta \omega_N$ data for one sky situation. In all investigated sky situations, we chose only such solar elevations that could have occurred in the onetime Viking habitats at the $61^\circ$ northern latitude of the main sailing route. Thus, the maximal solar elevation was $29^\circ$ for spring equinox and $52^\circ$ for summer solstice [38,40]. Sky situations above these elevation limits were removed from data evaluation. The gnomonic lines were calculated with the programme developed by Bernáth et al. [13].

(d) Calculation and visualization of data

After computation of uncertainty propagation, we got $12 \times 90 = 1080$ values of North uncertainty $\Delta \omega_N$ for each sunstone crystal at equinox and solstice and for forenoon and afternoon in the $90(\theta, \rho)$ groups of the selected 1080 sky situations. In figure 3, we show the weighted mean

$$\langle \omega_{max} \rangle = \frac{\sum_{i=1}^{N}(\omega_{max})_i(\delta \omega)_i^{-2}}{\sum_{i=1}^{N}(\delta \omega)_i^{-2}}$$  \hspace{1cm} (2.1)$$

and standard deviation

$$\Delta \omega_{max} = \frac{\sum_{i=1}^{N}[(\omega_{max})_i - \overline{\omega_{max}}]^2}{N}, \quad \overline{\omega_{max}} = \frac{\sum_{i=1}^{N}(\omega_{max})_i}{N}$$  \hspace{1cm} (2.2)$$

of peaks $\omega_{max}$ of North uncertainties for each elevation interval marked with rectangles, where $\overline{\omega_{max}}$ is the arithmetic mean of the $\omega_{max}$ values in the dataset. The horizontal length of a rectangle is $2\Delta \omega_{max}$ and the vertical length is uniform. We separately studied navigation in the forenoon and in the afternoon. We also calculated the weighted mean and standard error (s.e.)

$$\delta \omega_{max} = \sqrt{\frac{1}{\sum_{i=1}^{N}(\delta \omega)_i^{-2}}}$$  \hspace{1cm} (2.3)$$

averaged for the 12 values of $\Delta \omega_N$ in the 90 different $(\theta, \rho)$ groups. (2.1)–(2.3) were calculated for the calcite, cordierite and tourmaline sunstone crystals and for spring equinox and summer solstice. We visualized these data with a matrix in which each cell belongs to a given $(\theta, \rho)$ pair and contains the weighted mean $\langle \omega_{max} \rangle$ of North uncertainties, the values of which are coded
3. Results

Supplementary figure S2 shows the average $\langle \Delta \rangle \pm \text{s.d. } \sigma_\phi$ of the difference $\Delta = \phi_e - \phi_r$ between the estimated ($\phi_e$) and the real ($\phi_r$) azimuth angles of dots projected on a dome as a function of the
The uncertainty of this alignment was characterized by the measured $\sigma_{\text{sky-polarimetric navigation fitted to the measured}}$ function and for each Sun position $P_3$, we got an interval between $\phi_4(\theta) - \epsilon_{4\text{th}}(\theta)$ and $\phi_4(\theta) + \epsilon_{4\text{th}}(\theta)$ that was divided into equal horizontal angular distances by $0.2^\circ$ perpendicular to the meridian, where $\phi_4(\theta)$ is the azimuth angle of point $P_4$. Thus, the measured Sun position $P_4$ can be found in this inaccuracy interval. (h) Distribution (frequency $f$) of the North uncertainty $\Delta\omega_N$ with a maximum at angle $\omega_{\text{max}}$ and half bandwidth $\Delta\omega$ being the full width at half maximum. (i) The two possibilities to project the estimated sun position $E$ onto the forenoon and afternoon half of the gnomonic line. (j) The gnomonic lines for the spring equinox (21 March) and the summer solstice (21 June), onto which the real and estimated sun positions were projected. The angular deviation from the gnomonic lines gives the navigation uncertainty. $G$: gnomon. $S_E$: real Sun position projected onto the equinoctial line. $S'_E$: estimated erroneous Sun position projected onto the equinoctial line. $P_E$: point that we get after rotating $S'_E$ to fit onto the equinoctial line (the main step of North uncertainty computation). $S_S$: real Sun position projected onto the solstice line. $S'_S$: estimated erroneous Sun position projected onto the solstice line. $P_S$: point that we get after rotating $S'_S$ to fit onto the solstice line (the main step of North uncertainty computation). Grey: angles $\Delta\omega_N$ with which $S'_E$ and $S'_S$ need to be rotated to fit onto the equinoctial and solstice line, respectively. More details can be read in the text.

### Figure 2

Uncertainty function $\epsilon_{4\text{th}}(\theta)$ of the fourth step of sky-polarimetric navigation. The continuous curve is the parabola fitted to the measured $\epsilon(\theta)$ values.

Elevation angle $\theta$ of the dots measured psychophysically in a planetarium on 10 test persons 10-times. Figure 2 represents the uncertainty function $\epsilon_{4\text{th}}(\theta) = 0.0002\theta^2 + 1.1829$ of the fourth step of sky-polarimetric navigation fitted to the measured $\sigma_{\text{sky-polarimetric navigation fitted to the measured}}$ function.

According to figure 3 (electronic supplementary material, tables S1–S3), we can see that at summer solstice, the smallest North uncertainties occur at solar elevation $35^\circ \leq \theta \leq 40^\circ$ (with $\langle \omega_{\text{max}} \rangle = 3.4^\circ$ for calcite in the forenoon and $\langle \omega_{\text{max}} \rangle = 1.8^\circ$ in the afternoon, $\langle \omega_{\text{max}} \rangle = 2.9^\circ$ for cordierite in the forenoon and $\langle \omega_{\text{max}} \rangle = 1.6^\circ$ in the afternoon, $\langle \omega_{\text{max}} \rangle = 1.2^\circ$ for tourmaline in the forenoon and $\langle \omega_{\text{max}} \rangle = 0.2^\circ$ in the afternoon). At spring equinox, the smallest North uncertainties
Figure 3. Weighted mean $\langle \omega_{\text{max}} \rangle$ and s.d. $\Delta \omega_{\text{max}}$ of peaks $\omega_{\text{max}}$ of North uncertainties (errors) for each elevation interval marked with rectangles for the calcite, cordierite and tourmaline sunstone crystals for navigation in the forenoon and afternoon at spring equinox and summer solstice. The horizontal length of rectangles is $2 \Delta \omega_{\text{max}}$ and the $\langle \omega_{\text{max}} \rangle$ values are shown by the vertical bars in the centres of the rectangles. White and grey rectangles mean data for forenoon and afternoon, respectively.

The numerical values of these visualized data are in electronic supplementary material, tables S1 (calcite), S2 (cordierite) and S3 (tourmaline). The vertical dashed line is at $\Delta \omega_{\text{N}} = 0^\circ$.

occur for solar elevations $15^\circ \leq \theta \leq 25^\circ$ (with $|\langle \omega_{\text{max}} \rangle| = 6.2^\circ$ for calcite in the forenoon and $|\langle \omega_{\text{max}} \rangle| = 3.7^\circ$ in the afternoon, $|\langle \omega_{\text{max}} \rangle| = 7.1^\circ$ for cordierite in the forenoon and $|\langle \omega_{\text{max}} \rangle| = 3.0^\circ$ in the afternoon, $|\langle \omega_{\text{max}} \rangle| = 5.4^\circ$ for tourmaline in the forenoon and $|\langle \omega_{\text{max}} \rangle| = 6.3^\circ$ in the afternoon).

Using any of the three investigated sunstone crystals, at summer solstice, the s.d. $\Delta \omega_{\text{max}}$ of peaks $\omega_{\text{max}}$ of North uncertainties decreases with increasing solar elevation $\theta$ both for the forenoon and the afternoon navigation if $\theta < 35^\circ$, then $\Delta \omega_{\text{max}}$ increases strongly with increasing $\theta$ (figure 3). At spring equinox, however, $\Delta \omega_{\text{max}}$ tendentially increases with increasing $\theta$ for all three sunstones in the forenoon and in the afternoon (figure 3).

The North uncertainties both in the forenoon and the afternoon change sign (±) at $40^\circ \leq \theta \leq 45^\circ$ at summer solstice and at $20^\circ \leq \theta \leq 25^\circ$ at spring equinox (figure 3). The same effect occurs at spring equinox as well, though then the maximal solar elevation is lower.

Comparing the three sunstone crystals (figure 3; electronic supplementary material, tables S1–S3), at spring equinox for solar elevations $\theta \leq 20^\circ$, the smallest standard deviations $\Delta \omega_{\text{max}}$ of peaks $\omega_{\text{max}}$ of North uncertainties occur for calcite ($2.2^\circ < \Delta \omega_{\text{max}} < 4.3^\circ$), the largest $\Delta \omega_{\text{max}}$ values are for tourmaline ($2.5^\circ < \Delta \omega_{\text{max}} < 5.3^\circ$) and the $\Delta \omega_{\text{max}}$ values of cordierite are in-between ($2.1^\circ < \Delta \omega_{\text{max}} < 5.2^\circ$). At summer solstice for solar elevations $\theta \geq 35^\circ$, the smallest standard deviations $\Delta \omega_{\text{max}}$ of peaks $\omega_{\text{max}}$ of North uncertainties occur for calcite ($1.1^\circ < \Delta \omega_{\text{max}} < 4.5^\circ$), the largest $\Delta \omega_{\text{max}}$ values are for tourmaline ($2.1^\circ < \Delta \omega_{\text{max}} < 7.3^\circ$) and the $\Delta \omega_{\text{max}}$ values of cordierite are in-between ($1.4^\circ < \Delta \omega_{\text{max}} < 5.8^\circ$).

Figures 4–6 (electronic supplementary material, tables S4–S6) show the weighted mean $\langle \omega_{\text{max}} \rangle$ and s.e. $\delta \omega_{\text{max}}$ of peaks $\omega_{\text{max}}$ of North uncertainties for different sky situations characterized by the groups of solar elevation $\theta$ and cloudiness $\rho$:

— At summer solstice: the most accurate navigation is at $35^\circ \leq \theta \leq 40^\circ$ in the case of all three sunstone crystals (figures 4–6). Then, the most accurate navigation with minimal absolute weighted mean $|\langle \omega_{\text{max}} \rangle|$ ($|\langle \omega_{\text{max}} \rangle| = -1.4^\circ$) is at $\rho = 2$ and 7 oktas in the forenoon and at $\rho = 7$ oktas ($|\langle \omega_{\text{max}} \rangle| = 0.1^\circ$) in the afternoon for calcite (figure 4), at $\rho = 7$ oktas ($|\langle \omega_{\text{max}} \rangle| = -0.4^\circ$) in the forenoon and at $\rho = 2$ oktas ($|\langle \omega_{\text{max}} \rangle| = 0.0^\circ$) in the afternoon for
cordierite (figure 5), at $\rho = 3$ oktas ($\omega_{\text{max}} = -0.2^\circ$) in the forenoon and at $\rho = 3$ oktas ($\omega_{\text{max}} = 0.3^\circ$) in the afternoon for tourmaline (figure 6).

— At spring equinox: the minimum $|\omega_{\text{max}}|$ values are at $15^\circ \leq \theta \leq 20^\circ$, $\rho = 8$ oktas ($\omega_{\text{max}} = -0.2^\circ$) in the forenoon and at $20^\circ \leq \theta \leq 25^\circ$, $\rho = 1$ okta ($\omega_{\text{max}} = -0.4^\circ$) in the afternoon for calcite (figure 4), at $15^\circ \leq \theta \leq 20^\circ$, $\rho = 8$ oktas ($\omega_{\text{max}} = 0.1^\circ$) in the forenoon and at $20^\circ \leq \theta \leq 25^\circ$, $\rho = 0$ okta ($\omega_{\text{max}} = -0.1^\circ$) in the afternoon for cordierite (figure 5), and at $15^\circ \leq \theta \leq 20^\circ$, $\rho = 8$ oktas ($\omega_{\text{max}} = 0.9^\circ$) in the forenoon and at $20^\circ \leq \theta \leq 25^\circ$, $\rho = 1$ okta ($\omega_{\text{max}} = -1.1^\circ$) in the afternoon for tourmaline (figure 6). For a given $\rho$, the absolute value of $\omega_{\text{max}}$ dramatically decreases with increasing $\theta$ until the sign (±) change of $\omega_{\text{max}}$ values. For a given $\theta$, this absolute value can either decrease or increase with increasing $\rho$, meaning that there is no strong dependence on the cloud coverage $\rho$ (figures 4–6).

The most unsuitable meteorological situations for sky-polarimetric navigation are the following (figures 4–6):

(i) High solar elevations: $45^\circ \leq \theta \leq 50^\circ$ when $27^\circ \leq |\omega_{\text{max}}|_{\text{calcite}} \leq 47^\circ$ (figure 4), $23^\circ \leq |\omega_{\text{max}}|_{\text{cordierite}} \leq 47^\circ$ (figure 5), $26^\circ \leq |\omega_{\text{max}}|_{\text{tourmaline}} \leq 51^\circ$ (figure 6). These elevation values occur only at summer solstice.
The longitude could probably not have been determined by the Vikings, because this requires the availability of accurate clocks. The purpose of the Viking sun compass was to determine a specific reference direction (such as the North or the West), rather than the longitude. However, using this instrument, the longitude and local noon could also have been determined as Bernáth et al. [14] alternatively interpreted the Viking sundial artefact. Although Bernáth et al. [14] have also shown that the Uunartoq artefact fragment could also have been used before sunrise and after sunset, in this work we followed the original theory about the application of the device as a sun compass. Here, we determined the accuracy of sky-polarimetric navigation with the use of the earlier measured uncertainty functions of the four navigation steps for birefringent calcite and dichroic cordierite and tourmaline sunstone crystals [38–40] for 1080 different sky situations characterized by the solar elevation \( \theta \) and cloudiness \( \rho \). We obtained that both in the forenoon and the afternoon and for all three sunstones, at spring equinox the standard deviation of North uncertainty peaks \( \Delta \omega_{\text{max}} \) tendentiously increases with increasing \( \theta \), while at summer solstice the s.d. \( \Delta \omega_{\text{max}} \) of peaks \( \omega_{\text{max}} \) of North uncertainties decreases with increasing solar elevation \( \theta \) if \( \theta < 35^\circ \), then \( \Delta \omega_{\text{max}} \) increases strongly with increasing \( \theta \) (figure 3). The reason for these is the complex interaction of the \( \theta \)-dependent uncertainty functions of the four steps of sky-polarimetric navigation during the uncertainty propagation.

(ii) Low solar elevations with high cloudiness: \( 0^\circ \leq \theta \leq 10^\circ \), 5 oktas \( \leq \rho \leq 8 \) oktas. Then, the weighted mean of navigation uncertainties are \( 21^\circ \leq \langle \omega_{\text{max}} \rangle_{\text{calcite}} \leq 38^\circ \) (figure 4), \( 21^\circ \leq \langle \omega_{\text{max}} \rangle_{\text{cordierite}} \leq 39^\circ \) (figure 5) and \( 22^\circ \leq \langle \omega_{\text{max}} \rangle_{\text{tourmaline}} \leq 44^\circ \) (figure 6).

4. Discussion

The longitude could probably not have been determined by the Vikings, because this requires the availability of accurate clocks. The purpose of the Viking sun compass was to determine a specific reference direction (such as the North or the West), rather than the longitude. However, using this instrument, the longitude and local noon could also have been determined as Bernáth et al. [13] alternatively interpreted the Viking sundial artefact. Although Bernáth et al. [14] have also shown that the Uunartoq artefact fragment could also have been used before sunrise and after sunset, in this work we followed the original theory about the application of the device as a sun compass. Here, we determined the accuracy of sky-polarimetric navigation with the use of the earlier measured uncertainty functions of the four navigation steps for birefringent calcite and dichroic cordierite and tourmaline sunstone crystals [38–40] for 1080 different sky situations characterized by the solar elevation \( \theta \) and cloudiness \( \rho \). We obtained that both in the forenoon and the afternoon and for all three sunstones, at spring equinox the standard deviation of North uncertainty peaks \( \Delta \omega_{\text{max}} \) tendentiously increases with increasing \( \theta \), while at summer solstice the s.d. \( \Delta \omega_{\text{max}} \) of peaks \( \omega_{\text{max}} \) of North uncertainties decreases with increasing solar elevation \( \theta \) if \( \theta < 35^\circ \), then \( \Delta \omega_{\text{max}} \) increases strongly with increasing \( \theta \) (figure 3). The reason for these is the complex interaction of the \( \theta \)-dependent uncertainty functions of the four steps of sky-polarimetric navigation during the uncertainty propagation.
The net North uncertainty is the result of a complex propagation of the uncertainties of the four navigation steps. The uncertainties from the first step contribute considerably to the net navigation uncertainty if the degree of polarization $p$ of skylight is low. The uncertainties of the second step are the most dominant at low solar elevations ($\theta < 20^\circ$ for spring equinox) and the uncertainties of the third step become dominant at high solar elevations. The fourth step (the error of which is always less than 2.5°) has only a slight contribution to the net North uncertainty which is the largest for the highest solar elevations $\theta > 45^\circ$ at summer solstice (figure 2).

With low cloudiness, the direct sun could frequently be used for Viking navigation. Even with high cloud cover, the sky radiation and polarization are variable. To a given cloudiness infinite cloud patterns can belong, in many of which the sun is visible. However, in our 1080 carefully selected cloudy skies, the sun was occluded by clouds; furthermore, we selected 12 different skies with invisible sun for a given $\rho$-$\theta$ cell. Thus, although using oktas for the description of the celestial cloud cover is far from being complete, the 1080 different sky situations are enough to model the variability of the cloudy sky with which Viking navigators had to cope.

The sign change of the North uncertainty both in the forenoon and the afternoon (figure 3) can partly be explained with the characteristics of the elevation uncertainty in the third step of sky-polarimetric navigation: The contribution of underestimating solar elevation increases with increasing $\theta$ [40]; thus, the North uncertainties are shifted in the opposite direction. At lower solar elevations, the navigator can practically only overestimate the elevation $\theta$, because underestimations would cause the Sun positioned below the horizon that did not occur in our situations. In such cases (at lower solar elevations), there is no gnomon shadow that could have

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**Figure 6.** The plot same as figure 4 for tourmaline sunstone crystal. The numerical values of these data are in electronic supplementary material, table S6. (Online version in colour.)
reached the gnomonic line on the horizontal surface of the sun compass. However, elevation uncertainties for low solar elevations are much smaller than for high elevations [40].

The third step uncertainty cannot alone explain the sign change. The net uncertainty of the second navigation step [39] (electronic supplementary material, table S7) also has an elevation uncertainty component which is rather large (∣Δθ∣ < 21°) for low solar elevations (5° ≤ θ ≤ 25°) and is dominantly positive, meaning overestimations, while for high elevations (35° ≤ θ ≤ 55°), these uncertainties are small (∣Δθ∣ < 6°) and in approximately equal number positive and negative, which does not have a significant contribution to the resulting elevation uncertainty. At higher θ-values, as θ increases the overestimations of the solar elevation become less dominant, and for 35° ≤ θ ≤ 40° at summer solstice and for 20° ≤ θ ≤ 25° at spring equinox, over- and underestimations of θ occur with similar frequency. At high solar elevations around the daily maximum of θ, dominantly underestimations of θ can occur, because if the estimated Sun was above the possible daily maximum, the tip of the gnomon shadow could not reach the gnomonic line. In such cases, the Viking navigator had to re-measure the solar elevation, because he could not use it for North estimation. These underestimated solar elevations around noon give North uncertainties with the opposite sign relative to the overestimated elevations around sunset or sunrise. In figure 3, it is also clearly seen that logically, the sign of North uncertainties in the forenoon is the opposite of the sign in the afternoon, because the sun compass has to be rotated in the opposite direction, so that the shadow tip can reach the gnomonic line.

We obtained that sky-polarimetric navigation is most accurate for solar elevations 35° ≤ θ ≤ 40° at summer solstice and for 15° ≤ θ ≤ 25° at spring equinox (figures 4–6). It is remarkable that the majority of the less overcast situations were the most suitable (possessing the smallest North uncertainties) for sky-polarimetric navigation at a given solar elevation. However, we cannot exclude the possibility that this navigation method was also usable in strong cloudy circumstances, because in some sky situations the minimum of the weighted mean ∣⟨ωmax⟩∣ of North uncertainties is at cloudiness ρ = 7–8 oktas. We found that the most advantageous sky situations for this navigation method are at summer solstice when the solar elevation and cloudiness are 35° ≤ θ ≤ 40° and 1 okta ≤ ρ ≤ 7 oktas. Although there is no archaeological evidence of increased Viking seafaring during summertime (near summer solstice) as opposed to springtime (near spring equinox), our findings show that in summer, the sky-polarimetric navigation is more accurate than in spring.

The sky situations being the most unsuitable for sky-polarimetric navigation (i.e. having the largest North uncertainties) are more obvious: These are generally when the solar elevation is low (0° ≤ θ ≤ 10°) and the cloudiness is high (ρ = 5–8 oktas) when the orientation uncertainties are the highest for all three sunstone crystals and both for summer solstice and spring equinox. At summer solstice, very high solar elevations (45° ≤ θ ≤ 50°) can also be disadvantageous due to the high North uncertainties.

If the navigator measured and corrected his orientation several times a day with equal temporal distribution in the forenoon and the afternoon, the North uncertainties averaged for the whole day should be relatively small due to the opposite sign of forenoon and afternoon North uncertainties (figures 3–6). This is in accordance with the earlier findings [12] that in sunshine (when the Viking sun compass is easy to use in direct sunlight), it was worthwhile for the Vikings to orient themselves regularly, several times a day during their sailing routes.

Further uncertainties are introduced due to the different qualities of sunstones. Száz et al. [38] studied four calcite crystals of different qualities. In this work, we used the uncertainty function of the best calcite. Comparing the different sunstone crystal types, the use of birefringent calcite results in a more accurate sky-polarimetric navigation on average, because the weighted mean and standard error of North uncertainties are smaller than those for dichroic tourmaline and cordierite crystals. However, in some special sky situations (generally at 35° ≤ θ ≤ 40°, 1 okta ≤ ρ ≤ 6 oktas for summer solstice, and at 20° ≤ θ ≤ 25°, 0 okta ≤ ρ ≤ 4 oktas for spring equinox), tourmaline and cordierite crystals performed better, respectively, resulting in smaller North uncertainties, that is a more accurate navigation. The extraordinary performance of the calcite sunstone used
in this work can be explained by the fact that we selected the best-performing calcite crystal from the four different calcites used in our former psychophysical laboratory experiment [38]. Although this was a special choice, it is pertinent to suppose that Viking navigators might also have selected the best-performing calcite crystals from the available ones. If one crystal worked poorly during their journey (resulting in an inaccurate navigation), they could choose another one for their next journey, or, in the worst case, only those seafarers survived who had the best sunstones for navigation.

Most of the studies on Viking navigation [15,16,18,23–25] mentioned calcite crystals as the alleged Viking sunstones, without any quantitative measurements to prove their assumption. If a calcite crystal is ideal, having no contamination and crystal defects, it results in a more accurate navigation, because the two spots/slots seen through it due to double refraction ensure differential analysis of sky polarization, which is inherently more accurate than analysing polarization on the basis of the temporal sinusoid change of radiance of light transmitted through a rotating dichroic tourmaline or cordierite crystal. However, Száz et al. [38] showed that the adjustment uncertainties of calcite crystals with contamination and crystal defects can be larger than those of tourmaline and cordierite. Our results presented here corroborate experimentally the widespread belief that calcite can be a better sunstone than tourmaline and cordierite. But, we conclude that it is advisable to use calcite as sunstone under general sky conditions. In the above-mentioned special sky situations (generally at $35° \leq \theta \leq 40°, 1 \text{ okta} \leq \rho \leq 6 \text{ oktas}$ for summer solstice and at $20° \leq \theta \leq 25°, 0 \text{ okta} \leq \rho \leq 4 \text{ oktas}$ for spring equinox), it is worth choosing cordierite or tourmaline instead.

We interestingly found that a 61° North latitude sailing route may not have been as severe a hindrance to sky-polarimetric navigation as one would naively have thought, because north uncertainties dramatically increase for solar elevations $\theta > 40°$. Since the navigation uncertainty tendentiously increases with increasing $\theta$, a navigator based on the Equator with much larger $\theta$-values may have had quite a difficult time using this technique. Finally, we admit that in our studies (psychophysical laboratory and planetarium experiments), the circumstances were ideal. In a real-life situation, the continuous swaying and rolling of the ship, blowing and cold weather necessarily affect the accuracy of sky-polarimetric navigation. These real circumstances would have been a major handicap to align, for example, the shadow-stick or to accurately position the sunstones. Narrow vessels like those the Vikings used are prone to rocking and bobbing even in smooth seas, but can somewhat be stabilized by hoisting a sail. Thus, our results obviously underestimate the real-life North uncertainties of this navigation method.

Originally and most frequently, Vikings had determined the North (West–East) direction. Sometimes they had to know the actual latitude. If the latter differed considerably from 61°, then they had to compensate this by addition or subtraction of an angle to/from the North (West–East) direction. These are two different navigation tasks. The determination of the ideal frequency of the former task is the subject of a further study, the results of which will be published in a separate paper.

Although at night time the polar star could also have been used, in the time of Vikings the Polar star (Polaris) was positioned much farther from the celestial North Pole. Thus, Vikings could not have used it for accurate navigation.

This work deals with the solar elevations and cloudinesses favourable for the hypothetical sky-polarimetric Viking navigation. Using the results presented in this work, we plan to quantify the maximum navigation error that allows successful navigation meaning that voyages can reach the Viking settlement Hvarf in south Greenland from the Norwegian Hernam (now Bergen) along the 61° northern latitude, their main sailing route between Norway and Greenland. After such a computer simulation of the voyages, we can reveal how many degrees of navigation error throughout a voyage are acceptable. This could potentially be (i) an estimate of the latitudinal discrepancy resulting from a longitudinal voyage given (ii) a random walk of the azimuthal heading when exposed to the averaged peak North uncertainties and (iii) some average, representative skies across the duration of the voyage. The results of such a planned
investigation will demonstrate (i) how close to the planned disembarkation location a navigator could achieve, (ii) at what time of year and weather conditions is polarimetric navigation of highest and lowest quality, (iii) which sunstone type (calcite, cordierite or tourmaline) is the most favourable for this navigation and (iv) would polarimetric navigation truly have been a viable method of navigation for Viking seafarers.

Data accessibility. All supporting data of our studies are made available as the electronic supplementary material.

Authors’ contributions. Substantial contributions to conception and design: G.H., A.F. and D.S. Performing experiments and data acquisition: G.H., A.F., D.S., G.S., A.B., B.K., M.B. and Á.E. Data analysis and interpretation: G.H. and D.S. Drafting the article or revising it critically for important intellectual content: G.H., A.F. and D.S. All authors gave final approval for publication.

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References


47. Sharpe LT, Stockman A, Jagla W, Jagle H. 2005 A luminous efficiency function, $V^*(\lambda)$, for daylight adaptation. J. Vis. 5, 948–968. (doi:10.1167/5.11.3)