Psychophysical study of the visual sun location in pictures of cloudy and twilight skies inspired by Viking navigation

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In the late 1960s it was hypothesized that Vikings had been able to navigate the open seas, even when the sun was occluded by clouds or below the sea horizon, by using the angle of polarization of skylight. To detect the direction of skylight polarization, they were thought to have made use of birefringent crystals, called "sunstones," and a large part of the scientific community still firmly believe that Vikings were capable of polarimetric navigation. However, there are some critics who treat the usefulness of skylight polarization for orientation under partly cloudy or twilight conditions with extreme skepticism. One of their counterarguments has been the assumption that solar positions or solar azimuth directions could be estimated quite accurately by the naked eye, even if the sun was behind clouds or below the sea horizon. Thus under partly cloudy or twilight conditions there might have been no serious need for a polarimetric method to determine the position of the sun. The aim of our study was to test quantitatively the validity of this qualitative counterargument. In our psychophysical laboratory experiments, test subjects were confronted with numerous 180° field-of-view color photographs of partly cloudy skies with the sun occluded by clouds or of twilight skies with the sun below the horizon. The task of the subjects was to guess the position or the azimuth direction of the invisible sun with the naked eye. We calculated means and standard deviations of the estimated solar positions and azimuth angles to characterize the accuracy of the visual sun location. Our data do not support the common belief that the invisible sun can be located quite accurately from the celestial brightness and/or color patterns under cloudy or twilight conditions. Although our results underestimate the accuracy of visual sun location by experienced Viking navigators, the mentioned counterargument cannot be taken seriously as a valid criticism of the theory of the alleged polarimetric Viking navigation. Our results, however, do not bear on the polarimetric theory itself.

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1. INTRODUCTION

In the late 1960s Ramskou[1,2] hypothesized that Vikings had been able to navigate on the open sea by means of the angle of skylight polarization, even when the sun was occluded by clouds or below the sea horizon. The Vikings were thought to have detected the direction of skylight polarization with the help of a birefringent crystal, called "sunstone." This theory of polarimetric Viking navigation is accepted and frequently cited by a large part of the scientific community.[3–12]

However, Roslund and Beckman[13] on the basis of historical, archaeological, and practical sources summarized the lack of evidence for the hypothesis that Viking navigators used celestial polarization, and they highlighted the evidence for other information used by navigators. One of these other possible sources of information is the naked-eye estimation of the obscured sun position. According to their hypothesis, "Even when the Sun is hidden behind clouds, its location can often be found quite accurately for most navigational needs from the pattern of the Sun's illumination of clouds, from the bright lining of cloud tops and the crepuscular rays emanating from the Sun. On overcast days, careful observations of the sky may reveal the faint disk of the Sun if the cloud cover is not too dense. … Nor does polarimetry give clues to the Sun's position when it is below the horizon that other methods do not. The arcs of dawn and twilight appear distinct enough for the naked eye to make out in which direction the Sun is" (p. 4755). Unfortunately, Roslund and Beckman[13] did not define what they meant by "quite accurately." By "quite accurate location" they probably understood an angular error of a few degrees. Practically the same qualitative counterargumentation was repeated by Schaefer,[14] for example, "With partially cloudy or twilight conditions … the Sun's position is more easily apparent from the sky brightness distribution."

Yet the attractive and widely accepted theory of polarimetric Viking navigation cannot simply be refuted by such qualitative counterarguments. Quantitative studies are needed to test the various hypotheses of the theory and all the counterarguments of its opponents. As mentioned above, one of the counterarguments is the assumption that the sun position or the solar azimuth direction can be estimated quite accurately by the naked eye even if the sun is behind clouds or below the sea horizon and thus under partly cloudy or twilight conditions there might be no serious need for a polarimetric method to determine where the sun is.

The aim of this study was to test quantitatively the validity of this qualitative counterargument: In our psychophysical laboratory experiments, subjects were confronted with numerous 180° field-of-view color photographs of the partly cloudy sky with the sun occluded by clouds or of the twilight sky with the sun below the sea horizon. The task of the test subjects was then to guess the position or the azimuth direction of the invisible sun by the naked eye. We calculated means and standard deviations of the estimated solar positions and azimuth angles to characterize the accuracy of visual sun location. Our results do not support the above-mentioned counterargument, i.e., the common belief that the sun can usually be located quite accurately from the celestial brightness and color patterns under cloudy or twilight conditions. Note, however, that our results are opposed to only one of the counterarguments to the theory of polarimetric Viking navigation but by no means provide a definitive judgment on the polarimetric theory itself.

2. MATERIALS AND METHODS

In July 2001, color photographs of various cloudy skies were taken on the shore of the Finnish island Hailuoto (65°6′N,24°27′E) and the town of Oulu (65°0′N,25°26′E) with a Nikon F801 camera and a Nikon-Nikkor fisheye lens (*f*-number=2.8, focal length =8 mm) with a field of view of 180°. All relevant optical characteristics of this optical system are given in Ref. 15. The most important characteristic of the image formation of the Nikon-Nikkor fisheye lens is that the so-called projection angle is approximately the same as the incident off-axis angle (see Fig. 1C of Ref. 15). The consequence of this feature is that there is practically no radial angular deformation in the circular sky images used in our psychophysical experiments. Fujichrome Sensia II 100 ASA color reversal films were used. The optical axis of the fisheye lens was vertical and pointed at the zenith. Thus the full sky was recorded in the form of circular color pictures, in which the zenith was in the center and the horizon at the perimeter (Fig. 1). In another series of recordings the optical axis of the fisheye lens pointed at the sea horizon and photographs were taken when the sun was below the horizon but the twilight sky was still bright enough for visual inspection of the sunset or sunrise glow (Fig. 2). Following chemical development, these 180° field-of-view color photographs were digitized by a Hewlett Packard ScanJet 6100C with 8 (red)+8 (green)+8 (blue) bits (true color).

In our first psychophysical laboratory experimental series, color photographs of skies with the sun occluded by clouds (Fig. 1) were displayed on a color monitor (DTK Computer, 19-in.) in a dark room. The viewing distance from the subject to the stimulus was 30 cm, and the viewing angle subtended by the stimulus was 40°. All of our test subjects could accommodate to the sky pictures on the monitor; consequently, the 30-cm viewing distance was sufficient. Test subjects ($N=18$) had to click a mouse at the estimated position of the invisible sun located by the naked eye. One series of sky photographs consisted of $2\times 25=50$ pictures, involving 25 different cloudy skies (Fig. 1). Hence in the series a given sky was presented two times, and one of the two pictures was rotated around the zenith by a random angle. In one session of the experiment a given subject saw the series of 50 cloudy sky pictures two times with a break of 10 min. This procedure was repeated twice over an interval of a few days. Thus a given participant saw each of the 25 different cloudy skies $2\times 6=12$ times. To avoid an order effect, each of the 6 series of cloudy sky photographs had 6 different random orders, which were the same for all 18 observers. The computer program developed by us registered the estimated sun positions (θ, zenith angle; φ, azimuth angle measured from an arbitrary reference azimuth direction) and calculated their means ($\langle\theta\rangle,\langle\varphi\rangle$) and standard deviations ($\sigma_{\parallel},\sigma_{\perp},\sigma_{\varphi}$) with the following algorithm:

Consider the set of sun positions S_i, estimated visually by the subjects in relation to a given sky picture. In the Descartes system of coordinates of Fig. 3(a), the sky dome is represented by a unity hemisphere with a radius of $r=1$. The ith sun position S_i is represented by the unity vector r_i with polar angles θ_i and φ_i measured from axes Z and X, respectively [Fig. 3(a)]:

$$\mathbf{r}_i(\theta_i,\varphi_i)=(\sin\theta_i\cos\varphi_i,\sin\theta_i\sin\varphi_i,\cos\theta_i). \quad (1)$$

The average sun position S is represented by the unity vector $\mathbf{R}$, which is parallel to the sum of vectors $\mathbf{r}_i$:

$$\mathbf{R}(\theta_R,\varphi_R)=\sum_{i=1}^{i=K}\mathbf{r}_i \Bigg/ \left|\sum_{i=1}^{i=K}\mathbf{r}_i\right|, \quad (2)$$

where K is the total number of the estimated sun positions. S_i compose a more or less elongated set of points on the surface of the sky dome (Fig. 1). The standard deviations $\sigma_{\parallel}$ and $\sigma_{\perp}$ of S_i are calculated along two orthogonal great circles $GC_{\parallel}$ and $GC_{\perp}$ crossing each other at S [Fig. 3(b)]. $GC_{\parallel}$ and $GC_{\perp}$ are determined as follows: Using the angular distance α_i of $\mathbf{r}_i$ from the plane of an arbitrary great circle GC crossing S, we calculate the quantity

$$\sigma=\left(\sum_{i=1}^{i=K}\alpha_i^2 \Bigg/ K\right)^{1/2}. \quad (3)$$

Rotating GC about R, $GC_{\perp}$ is determined for which σ is maximal: $\sigma_{max}=\sigma_{\perp}$ [Fig. 3(b)]. Then $\sigma=\sigma_{\parallel}$ is calculated for $GC_{\parallel}$ being perpendicular to $GC_{\perp}$. Note that $\sigma_{\parallel}$ can never be larger than $\sigma_{\perp}$.

In our second psychophysical experiment, the color photographs of twilight skies with the sun below the sea horizon were displayed (Fig. 2) on the monitor in a dark room. Here again, the viewing distance from the subject to the stimulus and the viewing angle of the stimulus were 30 cm and 40°, respectively. The same test subjects ($N=18$) as in the first experiment had to click a mouse at the visually estimated azimuth direction of the invisible

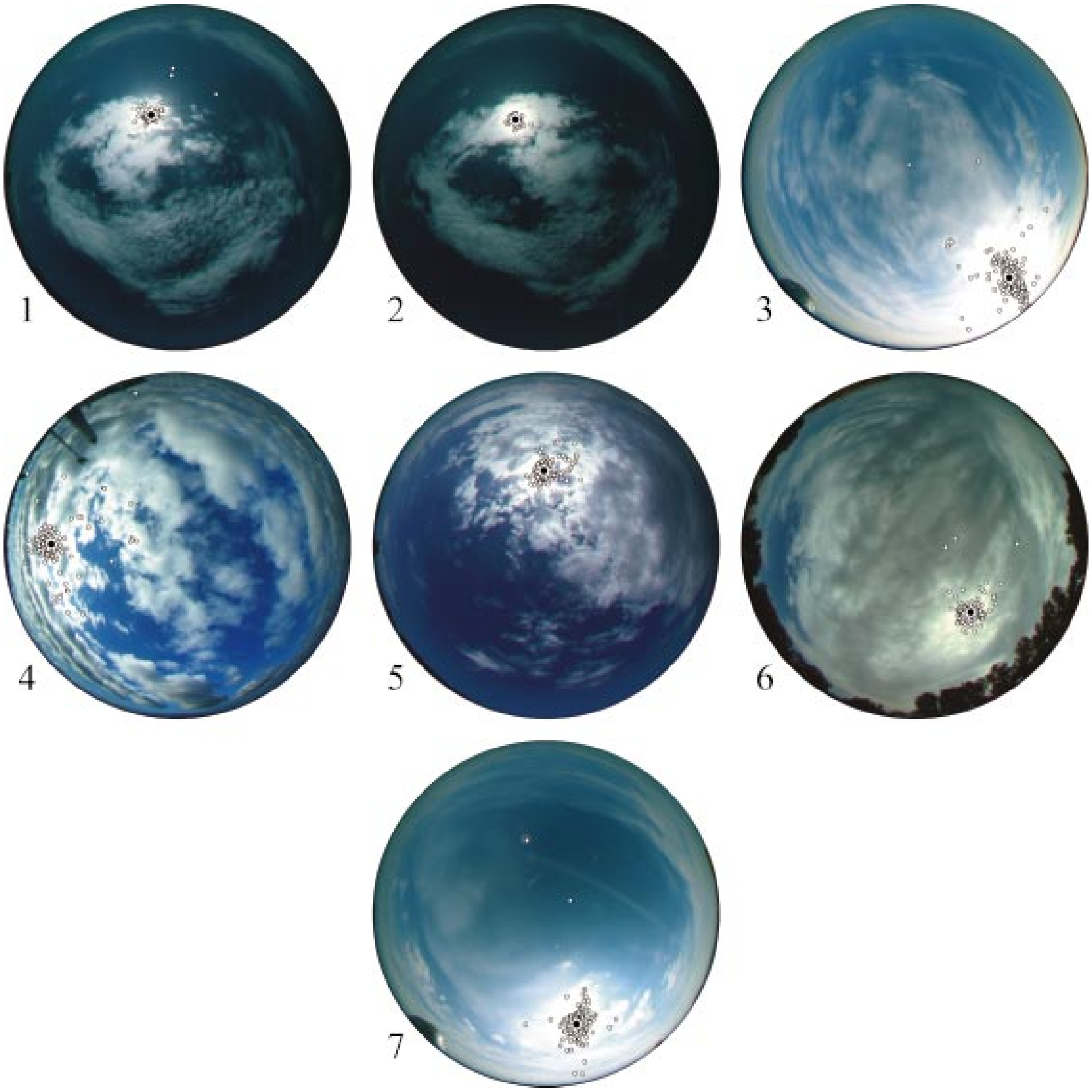

Fig. 1. Images of cloudy skies in which the invisible sun behind the clouds was located by the naked eye on a monitor in the first series of psychophysical laboratory experiments. The zenith is in the center of the circular sky pictures, and the horizon is at the perimeter. White dots outlined in black show the positions at which the test subjects expected to find the sun. The black dot in each picture represents the average of the solar positions (described by the zenith angles and the azimuth angles φ_i), the standard deviations $\sigma_\parallel$, $\sigma_\perp$, σ_φ of which are given in Table 1. The numbers of the sky pictures are the same as the numbers of the rows in Table 1. (Continues on next two pages.)

sun. A series of twilight sky photographs consisted of $3 \times 15 = 45$ pictures, involving 15 different twilight skies (Fig. 2). Hence in the series a given twilight sky was presented three times. In one session of the experiment a given participant saw the series of 3×15 twilight skies once, and the series was repeated after a few days. Thus a given subject saw the 15 different twilight skies 6 times. To avoid an order effect, each of the 2 series of twilight sky photographs had 2 different random orders, which were the same for all 18 observers. Our computer program registered the estimated solar azimuth angles φ and calculated their means $\langle\varphi\rangle$ and standard deviations σ_φ with the above-mentioned algorithm.

Our test subjects were naïve, navigationally untrained, urban men 23–45 yr of age living in Bremen, Germany; Budapest, Hungary; and Roskild, Denmark. They were recruited from the students and researchers of the universities of these cities and tested after ethical approval of the project. All subjects of our psychophysical experiments had good eyesight and did not wear eyeglasses. They had no great experience in guessing the location of the sun by the naked eye under cloudy and twilight conditions. Since our subjects were unfamiliar with circular full- or half-sky pictures, before the experiment they received instructions for the task to be performed. These preliminary experiments lasted 10 min, during which similar but cloudy and twilight sky pictures different from those in the real experiments were presented to the test subjects.

Since during photographing of the sky the exact orientation of the camera relative to the geographic north was not recorded, the exact solar positions and azimuth angles could not be calculated in the sky photographs, even if the geographic coordinates of the sites and the time of recordings were known. Thus the real solar positions and azimuth angles in the sky photographs used in our psycho-

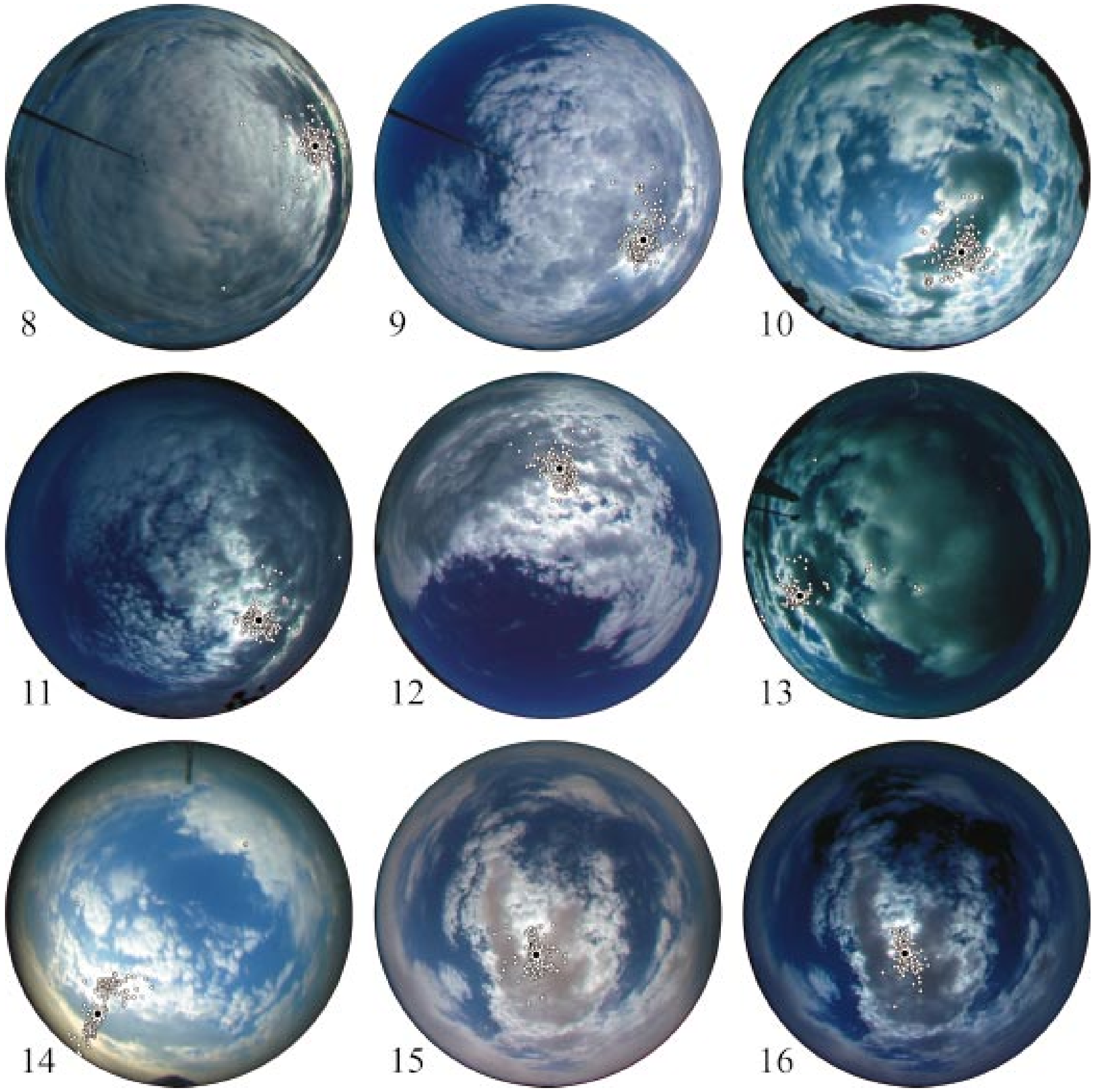

Fig. 1. (Continued).

physical experiments were unavailable. However, in Section 4 we explain why this lack of knowledge of the sun location is of no consequence with respect to our conclusion.

3. RESULTS

Figure 1 shows the 25 cloudy skies displayed on the monitor in our first experimental series. In these sky photographs all positions are also shown in which the invisible sun was located visually by the subjects. The standard deviations $\sigma_{\|}$, $\sigma_{\perp}$, σ_{φ} of the solar positions S_i (described by the zenith angle and the azimuth angle φ_i) are given in Table 1, which also gives the maximal angular distance $\delta_{\max}$ between the sun positions located in a given sky. Depending on the degree of cloud cover, standard deviations of the sun positions change from $\sigma_{\|}^{(\min)}=1.1°$, $\sigma_{\perp}^{(\min)}=1.4°$ (when the sun was nearly visible through a thin veil of cirrus cloud in pictures 1 and 2 of Fig. 1) to $\sigma_{\|}^{(\max)}=20.2°$, $\sigma_{\perp}^{(\max)}=25.2°$ (when the sun was covered by a thick layer of cloud). The maximal angular distances $\delta_{\max}$ between the estimated individual sun positions range from 8.1° to 162.9°. The means of $\sigma_{\|}$, $\sigma_{\perp}$, σ_{φ} and $\delta_{\max}$ averaged for all 25 cloudy skies were $\langle\sigma_{\|}\rangle=7.4°$, $\langle\sigma_{\perp}\rangle=11.9°$, $\langle\sigma_{\varphi}\rangle=22.3°$ and $\langle\delta_{\max}\rangle=70.7°$. According to Table 2, test subject 17 with $\langle\sigma_{\|}\rangle^{(\min)}=1.5°$, $\langle\sigma_{\perp}\rangle^{(\min)}=4.3°$, $\langle\delta_{\max}\rangle^{(\min)}=13.4°$ located the sun position with the smallest deviations, and subject 4 guessed the position of the sun with the highest errors of $\langle\sigma_{\|}\rangle^{(\max)}=5.6°$, $\langle\sigma_{\perp}\rangle^{(\max)}=15.3°$, $\langle\delta_{\max}\rangle^{(\max)}=50.4°$. The means of $\langle\sigma_{\|}\rangle$, $\langle\sigma_{\perp}\rangle$, and $\langle\delta_{\max}\rangle$ averaged for all 18 participants are $\langle\langle\sigma_{\|}\rangle\rangle=3.4°$, $\langle\langle\sigma_{\perp}\rangle\rangle=8.3°$, and $\langle\langle\delta_{\max}\rangle\rangle=24.8°$.

Figure 2 shows the 15 twilight skies displayed on the monitor in our second experimental series. These photographs show all directions at which the azimuth angle φ of the invisible sun below the sea horizon was located visually by the test subjects together with their averages $\langle\varphi\rangle$ and standard deviations σ_{φ}. The numerical values of σ_{φ} are given in Table 3. Table 3 also provides information on the maximal angular distance $\gamma_{\max}$ between the individual solar azimuths located in a given picture. In skies 1, 2, and 14 there are two distinct subpopulations of the estimated solar azimuths, for which σ_{φ} and $\gamma_{\max}$ are also calculated (Table 3) and represented (Fig. 2) separately.

Depending on the cloud cover of the twilight sky and

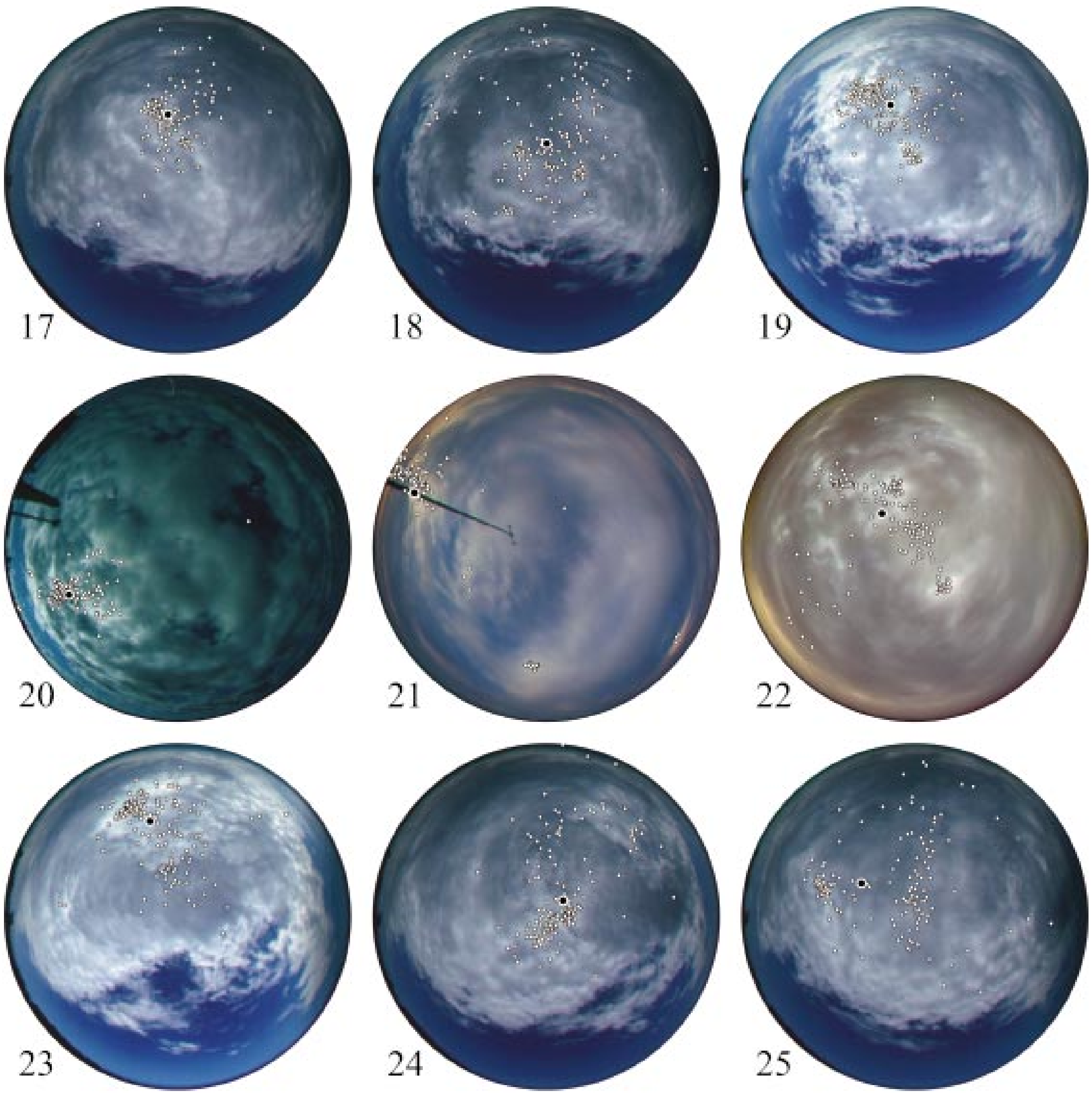

Fig. 1. (Continued).

the solar depression angle below the horizon, the standard deviations of the solar azimuths range from $\sigma_{\varphi}^{(\min)}=0.6°$ (when the sun was still visible at the horizon in picture 10 of Fig. 2) to $\sigma_{\varphi}^{(\max)}=42°$. The maximal angular distances $\gamma_{\max}$ between the estimated solar azimuths range from 2.1° (sun at the horizon, picture 10 in Fig. 2) to 99°. The means of σ_{φ} and $\gamma_{\max}$ averaged for all 15 twilight skies are $\langle\sigma_{\varphi}\rangle=11.4°$ and $\langle\gamma_{\max}\rangle=37.3°$. According to Table 4, test subject 3 with $\langle\sigma_{\varphi}\rangle^{(\min)}=2.4°$, $\langle\gamma_{\max}\rangle^{(\min)}=6.1°$ located the solar azimuth with the smallest deviations, and subject 2 with $\langle\sigma_{\varphi}\rangle^{(\max)}=11.2°$, $\langle\gamma_{\max}\rangle^{(\max)}=26.3°$ located the solar azimuth with the highest errors. The means of $\langle\sigma_{\varphi}\rangle$ and $\langle\gamma_{\max}\rangle$ averaged for all 18 participants are $\langle\langle\sigma_{\varphi}\rangle\rangle=5.9°$ and $\langle\langle\gamma_{\max}\rangle\rangle=14.5°$.

In Fig. 2 the majority of the pictures belong to five different series of recordings marked by A, B, C, D, and E. In Fig. 2 and Table 3, Δt is the time lag between consecutive pictures of a given series. In Table 3, $\Delta\varphi_{\Delta t}$ is the change of the solar azimuth angle during the period of Δt calculated with the computer program XEphem (http://www.clearskyinstitute.com/xephem). $\Delta\langle\varphi\rangle$ is the difference between the means of the visually detected azimuth angle $\langle\varphi\rangle$ of the consecutive pictures of a given series. The difference $\Delta\langle\varphi\rangle-\Delta\varphi_{\Delta t}$ defines how accurately the mean azimuth angle $\langle\varphi\rangle$ detected by the subjects follows the azimuth angle of the sun moving below the sea horizon in a given series. We note (Table 3) that in some cases of series A and C the change of the mean azimuth angle of the visually detected sun differs considerably $(9.9°\leqslant|\Delta\langle\varphi\rangle-\Delta\varphi_{\Delta t}|_{\max}\leqslant 11.7°)$ from the change of the true solar azimuth direction. On the other hand, in series B, D, and E the change of $\langle\varphi\rangle$ follows the change of the solar azimuth with significantly smaller errors $(2.9°\leqslant|\Delta\langle\varphi\rangle-\Delta\varphi_{\Delta t}|_{\max}\leqslant 5.6°)$.

4. DISCUSSION

Our aim was to investigate through simple psychophysical experiments one of the qualitative counter-

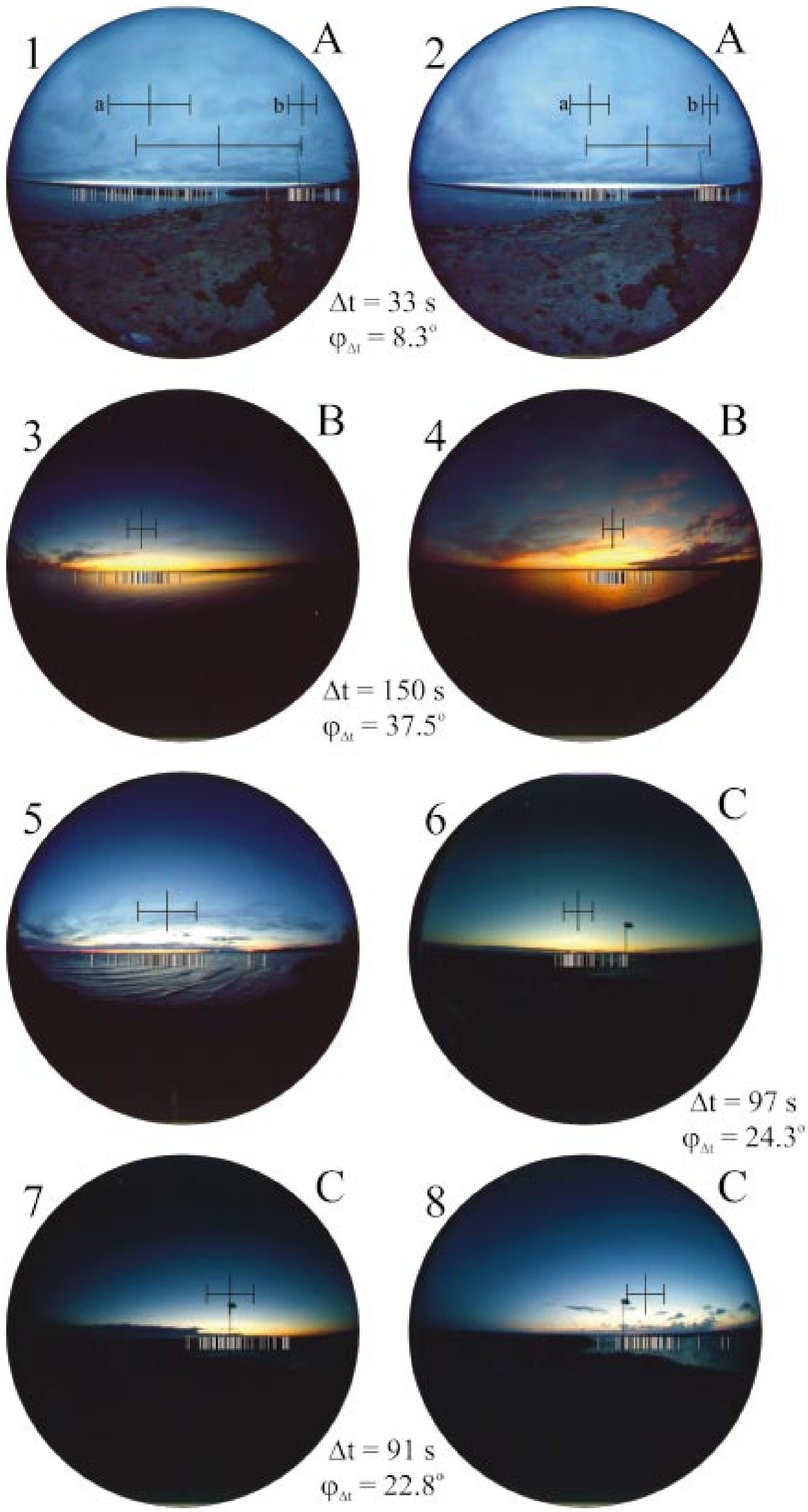

Fig. 2. Twilight skies in which the azimuth direction of the invisible sun below the sea horizon was located by the naked eye on a monitor in the second series of psychophysical laboratory experiments. The center of the circular pictures points to the horizon; the uppermost and lowermost points of the circle represent the zenith and the nadir, respectively. The upper half of the photographs is the sky; the lower part is the area below the sea horizon. Short white/black bars show all the directions below the horizon at which the azimuth angle of the invisible sun was thought to be located by the subjects. (Continues on next page.)

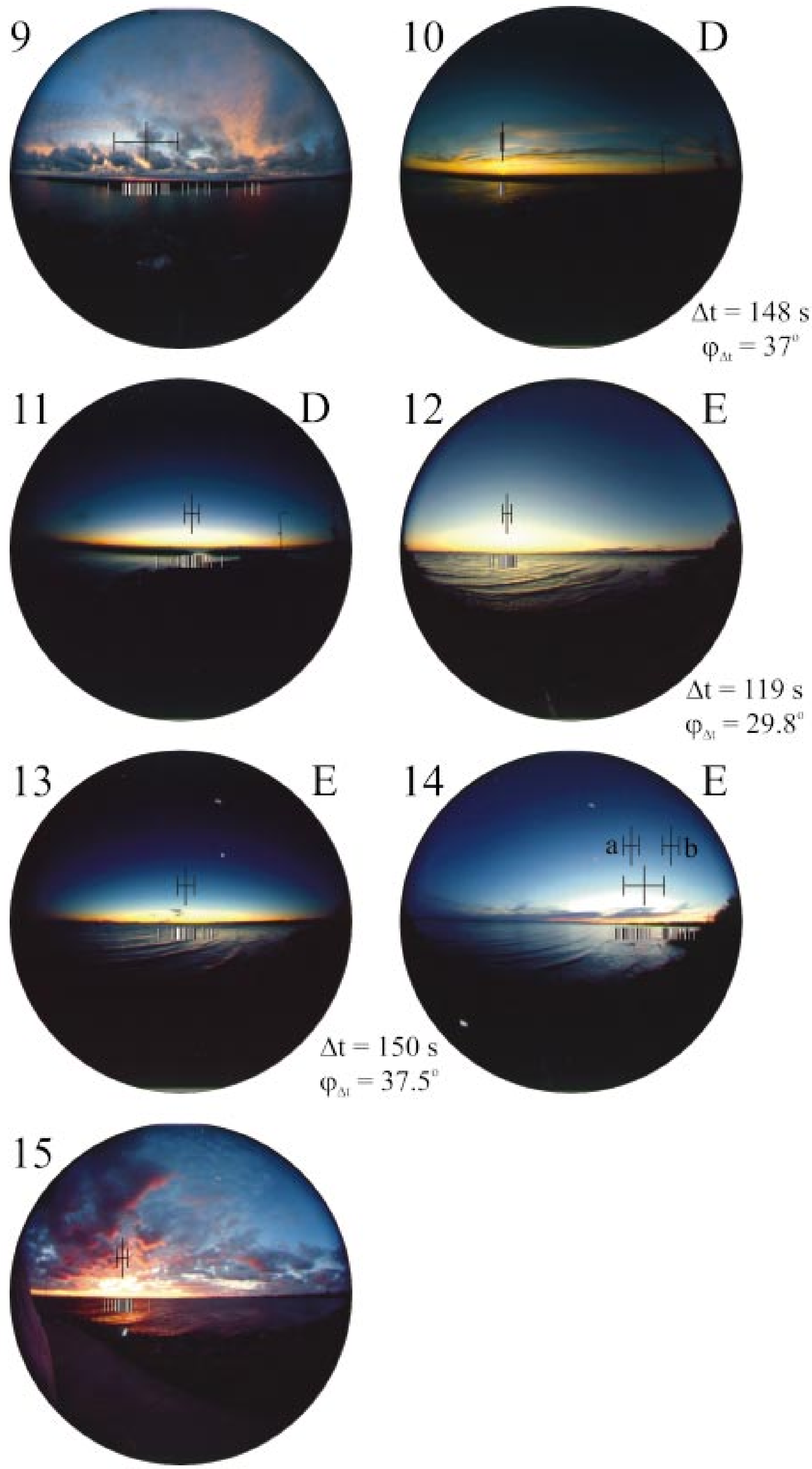

Fig. 2. *(Continued)*. The long vertical black bar above the horizon represents the average of the solar azimuth directions φ_i, the standard deviations of which ($\pm\sigma_\varphi$ in Table 3) are represented by short black vertical bars at the ends of the black horizontal bars. The numbers of the pictures are the same as the numbers of the rows in Table 3. Pictures belonging together (being members of the same series of recordings) are marked by the same capital letters (A, B, C, …). In skies 1, 2, and 14 there are two distinct subpopulations of the guessed solar azimuths, for which $\langle\varphi\rangle$ and σ_φ are also represented separately.

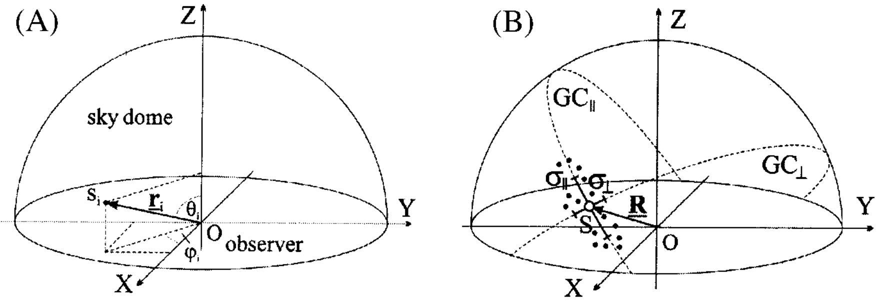

Fig. 3. Calculation of the mean $\underline{R}$ and standard deviations $\sigma_\parallel$, $\sigma_\perp$ of the visually estimated sun positions S_i in the sky dome (see Section 2). (A) The unity hemisphere as the representation of the sky dome with the unity vector $\underline{r}_i(\theta_i, \varphi_i)$ of the ith visually estimated sun position S_i. (B) The unity vector $\underline{R}$ of the average sun position S and the two orthogonal great circles $GC_\parallel$ and $GC_\perp$ (crossing S) along which the standard deviations $\sigma_\parallel$ and $\sigma_\perp$ were measured.

Table 1. Standard Deviations $\sigma_\parallel$, $\sigma_\perp$ of the Positions (Black Dots in Fig. 1) of the Invisible Sun behind Clouds Located with the Naked Eye 12 Times by 18 Subjects in Each Cloudy Sky of Fig. 1[a]

Cloudy Skies				
Picture No.	$\sigma_\parallel$ (°)	$\sigma_\perp$ (°)	δ_{max} (°)	σ_φ (°)
1	1.9	3.5	36.5	5.0
2	1.1	1.4	8.1	2.1
3	7.5	10.5	80.3	14.3
4	7.9	8.8	76.8	10.3
5	3.2	4.9	23.2	6.9
6	4.5	5.3	41.2	8.5
7	4.8	8.7	102.7	15.3
8	5.6	7.6	67.1	8.2
9	5.5	10.8	89.6	13.0
10	6.3	10.0	79.4	13.4
11	4.9	6.1	48.4	6.4
12	4.3	6.6	30.5	8.2
13	5.6	11.4	67.1	11.0
14	4.1	13.2	116.8	17.8
15	4.5	6.9	36.5	13.6
16	2.8	6.6	33.3	11.4
17	9.6	12.8	62.7	20.4
18	20.2	22.5	106.6	66.7
19	10.8	14.3	36.5	19.0
20	5.3	10.2	105.1	14.1
21	7.4	23.8	162.9	36.5
22	17.0	25.2	81.3	75.2
23	11.6	17.6	66.0	21.6
24	11.3	22.8	90.3	80.1
25	17.4	24.7	119.3	58.4
Average	$\langle\sigma_\parallel\rangle$	$\langle\sigma_\perp\rangle$	$\langle\delta_{max}\rangle$	$\langle\sigma_\varphi\rangle$
	7.4	11.9	70.7	22.3

[a] δ_{max} is the maximum angular distance between the sun positions located in a given sky. σ_φ is the standard deviation of the azimuth angles φ_i (measured from an arbitrary reference azimuth direction) of the estimated sun positions detected by the subjects. The row numbers in this table are the same as the numbers of the sky pictures in Fig. 1.

arguments[13,14] of the theory of polarimetric Viking navigation. The addressed counterargument is based on the hypothesis that the solar position or the solar azimuth can be visually estimated quite accurately even if the sun is behind clouds or below the sea horizon. (If this assumption were true, then there might not have been a serious need for a polarimetric method to help Viking navigators to guess the sun location under partly cloudy or twilight conditions.) Until now, this hypothesis has not been tested quantitatively. Our study is, to our knowledge, the first quantitative account addressing the accuracy of sun location with the naked eye in pictures of cloudy and twilight skies. Our results can be considered an underestimation of the accuracy of visual sun location, because in reality, Viking navigators could inspect the three-dimensional sky dome rather than two-dimensional pictures of the sky. For testing the mentioned hypothesis, ideal psychophysical experiments would fulfill the following conditions:

Table 2. Mean Standard Deviations $\langle\sigma_\parallel\rangle$ and $\langle\sigma_\perp\rangle$ of the Sun Positions and the Means $\langle\delta_{max}\rangle$ of the Maximal Angular Distances between the Sun Positions Located Visually 12 Times by a Given Subject (from 1 to 18) at a Given Cloudy Sky Averaged for the 25 Pictures in Fig. 1

Cloudy Skies			
Subject No.	$\langle\sigma_\parallel\rangle$ (°)	$\langle\sigma_\perp\rangle$ (°)	$\langle\delta_{max}\rangle$ (°)
1	2.5	5.4	15.3
2	4.6	9.9	30.9
3	3.4	6.9	23.2
4	5.6	15.3	50.4
5	3.3	9.6	27.3
6	2.5	6.6	19.7
7	3.0	6.3	19.7
8	1.7	4.9	14.6
9	3.9	9.3	23.8
10	4.8	14.7	43.6
11	3.3	7.1	19.7
12	4.7	11.3	33.2
13	3.0	7.5	20.5
14	2.3	6.7	18.6
15	3.9	7.1	21.6
16	1.5	7.2	21.5
17	1.5	4.3	13.4
18	5.4	9.3	29.4
Average	$\langle\langle\sigma_\parallel\rangle\rangle$	$\langle\langle\sigma_\perp\rangle\rangle$	$\langle\langle\delta_{max}\rangle\rangle$
	3.4	8.3	24.8

1. It would be advantageous if all subjects (e.g., experienced fishers or captains of sailing boats) were familiar with locating the sun by naked eye in cloudy or twilight skies.

However, nowadays in the era of accurate electronic global positioning systems, there are no longer any people who would frequently be confronted with the demand of sun location with the naked eye, i.e., without any instrument. Therefore everyone (including fishers or captains) can be considered a naïve subject considering this task. Thus it is not a limiting factor that the participants of our experiments were all naïve (i.e., untrained, urban men).

2. Full-cue information about the sky would be a favorable condition; i.e., it would be pertinent to perform our experiments under real skies.

However, the celestial cues would then be irreproducible, and each real sky should be presented simultaneously to all and the same subjects. This would mean nothing other than the need to assemble the same numerous subjects under several different real cloudy and twilight skies at various points in time on as many different occasions as the statistics require for measuring the accuracy of the subjects' visual sun location. This would have been overly difficult, if not impossible, to achieve for a large number of test subjects and could be done only with a limited number of subjects. However, that would have been insufficient for the statistical treatment.

3. Another possibility would have been to display reproducible, three-dimensional, full-sky images to the subjects in a planetarium.

On the one hand, although the time-consuming three-dimensional registration, reproduction, and projection of the constant starry image of the night-sky dome is routinely solved in every planetarium, it would be practically impossible to repeat this with numerous different cloudy and twilight full-sky images. On the other hand, owing to the reproducibility of the various sky images, the presentation of the series could be done separately with each subject. However, this would require extreme long rental and usage periods of the planetarium, which could be shortened only by drastically reducing the number of subjects, something that would again come at the expense of the statistical validity.

4. A further requirement would be the separate registration of the positions and directions of the invisible sun estimated by the subjects independently of one another.

This could be performed only in such a way that the subjects would be optically separated from one another (either by moving them beyond the distance of visibility from one another or by screening them with the aid of curtains).

Since it would have been overly difficult to perform the desired ideal experiment that would have fulfilled all of the above four conditions, we decided on the following compromise: In order to perform our psychophysical experiments with a sufficient number of participants, as well as to be able to present the same full-sky images to each one of them, we displayed 180° field-of-view color photographs on a monitor to the subjects in the laboratory. We admit that our method had the following inevitable limitations:

1. Looking at pictures is quite different from looking at real skies. Even with a monocular viewer, for example, there are many cues that reveal the flatness and finite distance of the picture surface. Colors in the picture tend to appear as surface colors rather than aperture colors. Note, however, that this is less true for the color pictures presented on a monitor and viewed in a dark room, as in our experiments.

2. Our sky pictures on the monitor had a reduced dynamic range and lower contrasts than the natural scene.

3. Certain color and intensity gradients of skylight cannot be detected unless the observer magnifies them (e.g., by looking at a reflection of the sky in the water surface rather than at the sky itself).[16]

4. In our experiments the field of view (40°) was different from that of the real scene (~90° vertically and ~180° horizontally for the two human eyes), and there was some metrical deformation owing to our 180° field-of-view photographic technique. For the small field of view the pattern of eye movements and integration of glances were different from what one might experience in a real scene.

5. Our twilight sky pictures deprived the observers of visibility of half of the sky.

6. In a real situation, an observer's sense of time of day could give him or her some help in finding an obscured sun, whereas time of day for our subjects bore no relation to the true position of the sun in the pictures.

In spite of these or similar methodological limitations, in psychophysics it is a commonly used method to present the relevant visual cues in the form of color pictures rather than confronting the subjects with real scenes. On the other hand, note that the limitations in points (2)–(4) rather strengthen our main conclusion (that the sun can usually be located inaccurately from the brightness and color patterns of cloudy or twilight skies), because in our sky pictures they make the visual sun location easier than in real skies. Thus, considering points (2)–(4), our experiments underestimate the error of sun location by the naked eye:

i. Because our sky photographs had a smaller dynamic range than the natural scene, the region around the sun in our cloudy pictures (Fig. 1/1–7) and the bow above the set sun on the horizon in our twilight pictures (Fig. 2/3–8, 10–15) appeared quite bright or were slightly overexposed. From the symmetry of the semicircular or bow-shaped form of these bright or overexposed regions, our subjects could guess the solar position and direction more easily than from the real skies: In the pictures the sun should be positioned somewhere in the vicinity of the center of the semicircular bright spot or near the vertical symmetry axis of the bright bow. Again owing to this lower dynamic range, in our sky photographs the color gradients were slightly magnified, which improved the location of the sun, since it helped to visually determine the above-mentioned center and symmetry axis.

Table 3. Standard Deviation σ_φ of the Azimuth Angles φ_i of the Guessed Sun Position below the Sea Horizon Located Visually 6 Times by 18 Subjects in Each Twilight Sky of Fig. 2[a]

Twilight Skies							
Picture No.	Series	σ_φ (°)	γ_{max}[b] (°)	Δt[c] (min)	$\Delta\varphi_{\Delta t}$[d] (°)	$\Delta\langle\varphi\rangle$[e] (°)	$\Delta\langle\varphi\rangle - \Delta\varphi_{\Delta t}$ (°)
1	A	42.0	99.0				
1/a		20.7	37.5	–	–	–	–
1/b		7.3	22.5				
2	A	31.6	87.6	33	7.6	13.9	+6.3
2/a		9.8	31.5	33	7.6	19.3	+11.7
2/b		3.8	17.4	33	7.6	2.9	−4.7
3	B	7.2	37.5	–	–	–	–
4	B	5.3	17.7	150	35.2	31.9	−3.3
5	–	14.8	55.2	–	–	–	–
6	C	7.5	28.8	–	–	–	–
7	C	12.0	37.2	97	22.6	24.7	+2.1
8	C	9.5	39.3	91	21.2	11.3	−9.9
9	–	16.8	59.4	–	–	–	–
10	D	0.6	2.1	–	–	–	–
11	D	3.9	21.9	148	34.5	37.4	+2.9
12	E	2.3	11.1	–	–	–	–
13	E	4.6	20.7	119	27.8	31.1	+3.3
14	E	10.6	27.9	150	35.5	41.1	+5.6
14/a		4.1	13.2				
14/b		4.3	13.5				
15	–	2.9	13.5	–	–	–	–
Average		$\langle\sigma_\varphi\rangle$	$\langle\gamma_{max}\rangle$				
		11.4	37.3				

[a] σ_φ, short black vertical bars at the ends of the black horizontal bars in Fig. 2; φ_i, azimuthal angles measured from an arbitrary reference azimuth direction—long black vertical bars above the horizon of pictures in Fig. 2.

[b] Maximal angular distance between the solar azimuth directions located in a given sky.

[c] Time lag between the consecutive pictures of a given series (A, B, C, D, E).

[d] Change of the solar azimuth angle between consecutive pictures calculated with the computer program XEphem (http://www.clearskyinstitute.com/xephem).

[e] Difference between mean azimuth angles $\langle\varphi\rangle$ of consecutive pictures of a given series. Row numbers are the same as picture numbers in Fig. 2. In skies 1, 2, and 14 there are two distinct subpopulations (*a* and *b*) of the guessed solar azimuths, for which σ_φ, γ_{max} (and $\Delta\langle\varphi\rangle$ for picture 2) have been calculated separately.

ii. Because of the smaller field of view of our sky pictures, the eye movements were more limited and the integration of glances was easier, which made the visual sun location easier than in the real scenes. In other words, it is less difficult to comprehend a circular full-sky photograph and locate the sun on it than to scan the whole real sky, looking for the sun.

If the sun-occluding clouds are thin, there is only one bright white patch in the sky around the invisible sun. The sun can then be relatively easily located (e.g., skies 1, 2, and 7 in Fig. 1), and the standard deviations $\sigma_\parallel$, $\sigma_\perp$ of the solar positions are small (Table 1). The thicker and the larger the sun-occluding cloud, the more difficult it becomes to guess the solar position, and thus the larger are $\sigma_\parallel$ and $\sigma_\perp$ (e.g., skies 3–6 in Fig. 1; Table 1).

If the sun is behind a thick (dark) and substantial cloud, it can be located on the basis of the brightness pattern of the cloud perimeter: The closer the sun to the edge of the cloud, the brighter the margin there and the easier the location of the sun (e.g., skies 8–16 in Fig. 1). In this case the accuracy of the sun location is determined by the dimensions of the occluding cloud (Table 1).

If the cloud cover is thick and extensive, there may be several bright patches where the cloud layer is thinner. The sun can then be located in these brighter patches (e.g., skies 22–24 in Fig. 1). In this case the accuracy of sun location (determined primarily by the distance of the bright patches) is low; i.e., the standard deviations $\sigma_\parallel$, $\sigma_\perp$ of the solar position are large (Table 1).

In real skies the crepuscular rays (the bright and dark beams apparently radiating from the sun when blocked by clouds) help one considerably to guess where the sun is, because they cross one another at the solar position. This phenomenon makes it very easy to locate the sun. In spite of the reduced dynamic range and lower contrasts of our sky photographs compared with those of the natural scene, the crepuscular rays were not lost. However, for our experiments we selected only such sky pictures in which crepuscular rays did not occur, since these rays would have made the location of the sun extremely easy.

At twilight, the azimuth direction of the invisible sun below the horizon can be estimated on the basis of the following visual cues: the color and radiance pattern of the clear blue sky (e.g., pictures 6 and 11 in Fig. 2), the reddish-orange glow of the sky immediately above the horizon (e.g., pictures 3 and 13 in Fig. 2), the color and brightness patterns of clouds (if any) near the horizon (e.g., pictures 4 and 15 in Fig. 2), and the reflection pat-

Table 4. Mean Standard Deviation $\langle\sigma_\varphi\rangle$ of the Solar Azimuth Angles and Means $\langle\gamma_{max}\rangle$ of the Maximal Angular Distances between Solar Azimuths Located Visually 6 Times by a Given Subject (from 1 to 18) at a Given Twilight Sky, Averaged for the 15 Pictures in Fig. 2

	Twilight Skies	
Subject No.	$\langle\sigma_\varphi\rangle$ (°)	$\langle\gamma_{max}\rangle$ (°)
1	4.6	10.8
2	11.2	26.3
3	2.4	6.1
4	3.8	10.6
5	8.1	18.1
6	5.9	16.6
7	5.8	13.4
8	2.7	6.7
9	5.0	12.1
10	5.2	12.8
11	4.4	11.9
12	8.1	21.7
13	6.4	15.9
14	10.1	23.8
15	4.4	12.6
16	4.6	9.2
17	5.3	11.7
18	8.7	20.9
Average	$\langle\langle\sigma_\varphi\rangle\rangle$	$\langle\langle\gamma_{max}\rangle\rangle$
	5.9	14.5

tern of the sea surface (e.g., pictures 4 and 12 in Fig. 2). From Fig. 2 as well as Tables 3 and 4 it is clear that at twilight, when the sun is below the horizon and lower than ~2°, its location is difficult to ascertain. Although a bright and colored twilight arch can be seen, it occupies a large part of the horizon and is of relatively uniform intensity. The same holds true for the reflection pattern at the sea surface. A similar effect may conceivably occur when the sun is above but in the immediate vicinity of the horizon and a thick layer of cloud covers it.

From Tables 1 and 3 we can determine that the standard deviation of the sun positions estimated by the test persons of our psychophysical experiments is ~22° for full-sky (cloudy) pictures and ~11° for half-sky (twilight) pictures. If the vague term "quite accurately" used by Roslund and Beckman[13] means an error of some degrees, these errors do not support the common belief that the invisible sun can usually be located with the naked eye quite accurately from the celestial brightness and color patterns under cloudy or twilight conditions. Only further research can reveal the possible influence of these errors on Viking navigation under cloudy or twilight skies. Note, however, that knowledge of the solar azimuth angle alone is insufficient but useful for Viking navigation.[12,13]

All participants in our experiments live in big cities (Bremen, Budapest, Roskild) and have no great experience in guessing the location of the sun by the naked eye under cloudy and twilight conditions. Since the Viking navigators might have had several decades of experience in this task, our psychophysical studies underestimated the accuracy of the Vikings' visual sun location. In other words, one can easily imagine that experienced Viking navigators might have been considerably better at this task than our naïve subjects. The visual sun location could be considered accurate only if (a) the standard deviations $\sigma_\parallel$, $\sigma_\perp$ of the sun positions under cloudy conditions and the standard deviation σ_φ of the solar azimuths under twilight conditions were small, and (b) the average sun positions and solar azimuths differed only slightly from the real ones. Since according to our results condition (a) is not satisfied, our subjects located the sun inaccurately. This conclusion is not weakened by the fact that the real solar positions and azimuth angles in the sky pictures used in our psychophysical experiments were unknown and that therefore we cannot state anything about the satisfaction of condition (b).

Although our sky photographs were taken from slightly farther north (65° latitude) than one of the Vikings' most frequently used maritime routes at 61° N (between Hernam, on the West coast of Norway, and Hvarf, north of the southern tip of Greenland[12]), this cannot cause any problems, because for about 300 years the Vikings ruled the seas of a huge geographical region, the range of which involved both 61° and 65° of latitude, and in Finland (where our sky photographs were taken), the sky conditions (e.g., frequency and type of clouds) are practically the same as in the region covered by the Vikings between 61° and 65° of latitude.

Situations that produced the worst performance of our test subjects were those in which the sky and the sun were completely obscured by clouds or in which there were several bright spots in the clouds. Note that in both cases the hypothetical sunstone would also have been ineffective. In such situations a Viking navigator should have had to guess the sun position either with the naked eye or should have had to wait for the sun to be revealed. It is guesswork what a navigator would have done in this case. It is similarly unknown whether the costs of waiting (sometimes for several days) would have been greater in the open sea than the benefits of making some judgment on the position of the sun obscured by clouds.

Finally, we would like to emphasize that the relationship of the present study to the issue of whether the Vikings used skylight polarization as an aid is very indirect: We do not criticize the hypothesis of the polarimetric Viking navigation but only one of its counterarguments.

5. CONCLUSION

The measure of accuracy of visual sun location in cloudy skies with the sun behind clouds is defined by the standard deviations $\sigma_\parallel$, $\sigma_\perp$, σ_φ of the sun positions and the maximal angular distance δ_{max} between the estimated sun positions. Similarly, the measure of accuracy of visual sun location in twilight skies with the sun below the sea horizon is determined by the standard deviation σ_φ of the solar azimuth angles and the maximal angular distance γ_{max} between the estimated solar azimuth directions. The means of these variables, averaged for all investigated cloudy $(\langle\sigma_\parallel\rangle \approx 7°, \langle\sigma_\perp\rangle \approx 12°, \langle\sigma_\varphi\rangle \approx 22°, \langle\delta_{max}\rangle \approx 71°)$ and twilight $(\langle\sigma_\varphi\rangle \approx 11°, \langle\gamma_{max}\rangle \approx 37°)$ skies or averaged for all subjects (cloudy: $\langle\langle\sigma_{\parallel}\rangle\rangle\approx 3°$, $\langle\langle\sigma_{\perp}\rangle\rangle\approx 8°$, $\langle\langle\delta_{\max}\rangle\rangle\approx 25°$; twilight: $\langle\langle\sigma_{\varphi}\rangle\rangle\approx 6°$, $\langle\langle\gamma_{\max}\rangle\rangle\approx 15°$) are relatively high. The highest registered values are $\sigma_{\parallel}^{(\max)}\approx 20°$, $\sigma_{\perp}^{(\max)}\approx 25°$, $\sigma_{\varphi}^{(\max)}\approx 80°$, $\delta_{\max}^{(\max)}\approx 163°$ for the cloudy sky and $\sigma_{\varphi}^{(\max)}\approx 42°$, $\gamma_{\max}^{(\max)}\approx 99°$ for the twilight sky. These data do not support the common belief that the sun can usually be located quite accurately from the celestial brightness and color patterns under cloudy or twilight conditions. Although these results underestimate the accuracy of visual sun location by experienced Viking navigators, the mentioned counterargument cannot seriously challenge the theory of the alleged polarimetric Viking navigation.

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