applied optics

Polarization optics of the Brewster's dark patch visible on water surfaces versus solar height and sky conditions: theory, computer modeling, photography, and painting

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When the sun is near the horizon, a circular band with approximately vertically polarized skylight is formed at 90° from the sun, and this skylight is only weakly reflected from the region of the water surface around the Brewster's angle (53° from the nadir). Thus, at low solar heights under a clear sky, an extended dark patch is visible on the water surface when one looks toward the north or south quarter perpendicular to the solar vertical. In this work, we study the radiance distribution of this so-called Brewster's dark patch (BDP) in still water as functions of the solar height and sky conditions. We calculate the pattern of reflectivity *R* of a water surface for a clear sky and obtain from this idealized situation the shape of the BDP. From three full-sky polarimetric pictures taken about a clear, a partly cloudy, and an overcast sky, we determine the *R* pattern and compose from that synthetic color pictures showing how the radiance distribution of skylight reflected at the water surface and the BDPs would look under these sky conditions. We also present photographs taken without a linearly polarizing filter about the BDP. Finally, we show a 19th century painting on which a river is seen with a dark region of the water surface, which can be interpreted as an artistic illustration of the BDP.

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1. INTRODUCTION

If partially vertically polarized light hits the horizontal water surface at the Brewster's angle $\theta_{\text{Brewster}} = \arctan(m) = 53°$, where $m = 1.33$ is the refractive index of water, and θ_{Brewster} is measured from the vertical, its vertically polarized component is not reflected. Thus, if unpolarized light is reflected from the Brewster's angle, it becomes totally horizontally polarized, because, then, only the horizontally polarized component is reflected. Due to this phenomenon, at low solar heights and under a clear sky, an extended dark patch is visible on the water surface when one looks toward the north or south quarter perpendicular to the solar–antisolar vertical at the Brewster's angle [1,2] [Fig. 1(A)]. The reason for this is that when the sun is near the horizon, a circular band with an approximately vertically polarized skylight is formed at 90° from the sun [3,4] [Fig. 1(A)], and this skylight is not or only weakly reflected from the region of the water surface around the Brewster's angle, while outside of this dark patch, the bright skylight is progressively reflected with an increasing angle from the Brewster's direction.

This so-called Brewster's dark patch [BDP, Fig. 1(B)] can be photographed [1,2] and may be visible in various photographs. Although the reflection polarization of light from the water surface has been thoroughly studied [5], the polarization optics of the BDP have been neglected. Photographers often use a polarizer in front of their camera (i) to filter disturbing reflections or (ii) to obtain an aesthetic effect (using a vertical linear polarizer to darken watery foregrounds, for example). However, they usually do not communicate the information as to whether such a polarization filter was or was not applied during exposure. Thus, it is generally not known whether a darker area on a photo displaying a water surface is really the BDP, or if it

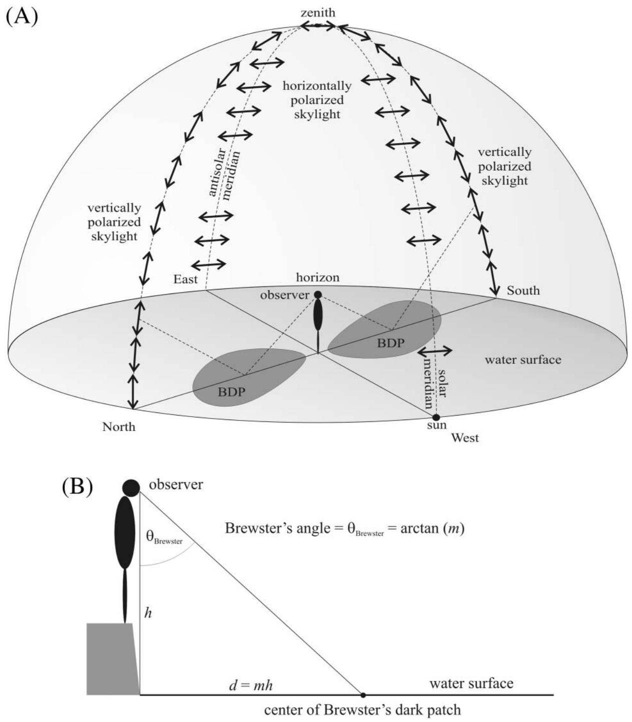

Fig. 1. (A) Geometry of the polarization optics of the Brewster's dark patch (BDP) visible on the water surface toward the north or south quarter when the sun is on or near the horizon. Double-headed arrows represent the horizontally polarized light coming from the solar–antisolar vertical of the celestial hemisphere. From the north–south vertical, being perpendicular to the solar–antisolar vertical, vertically polarized skylight originates. (B) Calculation of the horizontal distance $d = mh$ of the center of BDP from an observer viewing at the water surface from height h with an angle of view $\theta_{\text{Brewster}} = \arctan(m) = 53°$ relative to the vertical, where $m = 1.33$ is the refractive index of water.

originates from the partial or total blocking of horizontally polarized water-reflected light by a polarizer with a nearly or exactly vertical transmission axis. On these photos, both the solar height from the horizon and the direction of view of the camera relative to the solar vertical cannot often be deduced, apart from the trivial case when the sun is visible on them. However, the direction of view of the camera relative to the solar vertical can in many cases be inferred from the shadows cast by trees or other objects. Shadows of trees or buildings, for example, may also be mistaken for the BDP. Of course, the sky conditions (clear or cloudy, resulting in higher or lower degrees of skylight polarization) may also not be revealed if the sky is not seen in a photo with a water surface.

Since the BDP is a striking optical phenomenon frequently observable near sunrise and sunset when looking at a water surface toward the north or south quarter normal to the solar vertical, artists can also pick up on and illustrate it on their paintings displaying lakes or rivers. In the last decades, we intensively searched paintings with lakes or rivers and found some good candidates, which may represent the BDP on the water surface. The aims of this work are to (i) study the polarization optics of the BDP as functions of the solar height and sky conditions (cloudiness), (ii) present photographs taken without a linearly polarizing filter about the BDP visible on water surfaces when looking perpendicular to the solar vertical at low solar heights near sunset or sunrise, and (iii) show and environmentally optically interpret a painting illustrating a dark region of a river surface.

2. MATERIALS AND METHODS

A. Modeling the Shape of the Brewster's Dark Patch

The theoretical shapes of the BDP are obtained in the following way: as input, we used the theoretical spatial distributions of the radiance I, degree of linear polarization p, and angle of polarization α of the clear sky as calculated from the model of Berry *et al.* [6] as a function of the solar height h_S measured from the horizon. The parameters of this model were the following [7]: (i) maximum degree of polarization $p_{\max} = 50\%$ at 90° from the sun. (ii) The angular distance Δ between the Babinet's and Brewster's neutral (unpolarized) points was $\Delta(h_S = 0°) = 20°$ for solar height $h_S = 0°$, and it decreased linearly to $\Delta(h_S = 90°) = 0°$ with increasing h_S.

The I, p, and α patterns of real (clear, cloudy, overcast) skies were measured in the red (650 nm), green (550 nm), and blue (450 nm) parts of the spectrum by full-sky imaging polarimetry, the method of which is described in detail elsewhere [3].

Using Stokes–Mueller calculations of light reflection without the circular polarization component, the patterns of the degree and angle of polarization of the skylight reflected from the smooth (not undulating) water surface, as well as the pattern of the reflectivity of the water surface, were calculated as follows: the Stokes vector of the incident skylight is

$$S^{\text{skylight}} = I^{\text{skylight}}[1, -p \cdot \cos(2\alpha), p \cdot \cos(2\alpha), 0], \quad \textbf{(1)}$$

where I^{skylight} is the radiance, p is the degree of linear polarization, and α is the angle of polarization of the skylight measured from the vertical passing through the celestial point observed, i.e., the direction of the vertical was the reference direction for the Stokes vector [Fig. 1(A)]. The Mueller matrix of the air–water interface is

$$\underline{\underline{M}} = \frac{1}{2}\left(\frac{\tan\theta_-}{\sin\theta_+}\right)^2 \begin{pmatrix} \cos^2\theta_- + \cos^2\theta_+ & \cos^2\theta_- - \cos^2\theta_+ & 0 & 0 \\ \cos^2\theta_- - \cos^2\theta_+ & \cos^2\theta_- + \cos^2\theta_+ & 0 & 0 \\ 0 & 0 & -2\cos\theta_-\cos\theta_+ & 0 \\ 0 & 0 & 0 & -2\cos\theta_-\cos\theta_+ \end{pmatrix},$$

$$\theta_- = \theta_i - \theta_r, \qquad \theta_+ = \theta_i + \theta_r, \quad \textbf{(2)}$$

where θ_i and θ_r are the angles of incidence and refraction, respectively, measured from the vertical. Using the Snellius–Descartes law of refraction, we get

$$\theta_r = \arcsin\left(\frac{\sin\theta_i}{m}\right), \tag{3}$$

where $m = 1.33$ is the refractive index of water for wavelength $\lambda = 550$ nm. The Stokes vector of the reflected skylight is

$$\underline{S}^{\text{reflected}} = \underline{\underline{M}}\,\underline{S}^{\text{skylight}}, \tag{4}$$

from which the polarization parameters of the water-reflected skylight are the following:

$$I_{\text{reflected}} = S_0^{\text{refl}},$$

$$p_{\text{reflected}} = \sqrt{\frac{S_1^{\text{refl}^2} + S_2^{\text{refl}^2} + S_3^{\text{refl}^2}}{S_0^{\text{refl}^2}}} \approx \sqrt{\frac{S_1^{\text{refl}^2} + S_2^{\text{refl}^2}}{S_0^{\text{refl}^2}}},$$

because

$$S_3^{\text{refl}} \approx 0, \alpha_{\text{reflected}} = \frac{1}{2}\arctan\left(\frac{S_2^{\text{refl}}}{-S_1^{\text{refl}}}\right), \tag{5}$$

where S_n^{refl} ($n = 0, 1, 2, 3$) is the nth element of the Stokes vector. The reflectivity $R = I_{\text{water}}/I_{\text{skylight}}$ of the water surface was computed from pixel to pixel of the picture, where I_{water} and I_{skylight} are the radiance of the water-reflected skylight and the incident skylight, respectively.

The pattern of reflectivity $R = I_{\text{reflected}}/I_{\text{incident}}$ of the water surface was calculated for different theoretical and real (clear, cloudy, overcast) skies. We defined the Brewster's point as the position on the water surface with minimal reflectivity R_{min}. Deriving from this, the BDP is defined here as the region on the water surface with reflectivities not larger than a threshold of $R^*: R \leq R^*$. We put the value of the threshold R^* at 2%.

B. Construction of Synthetic Pictures of the Brewster's Dark Patch

The view an observer can see when looking perpendicular to the solar–antisolar vertical at the water-reflected mirror image of the sky is presented in the form of synthetic pictures. The synthetic pictures of the subhorizon views were obtained in the following way: we took a polarimetric color photograph with a 180° field-of-view fisheye lens. If the radiance of the skylight was too small, we performed a five times brightening, meaning that the radiance I was multiplied by five. Then, we calculated the mirror image of the sky by computing the radiance I_{water} of water-reflected skylight in the red (650 nm), green (550 nm), and blue (450 nm) spectral ranges, and, from these I_{water}-values, we composed a color picture seen on the water surface. Since this mirror image of the sky was always too dark, we applied a 50 times brightening, i.e., the radiance I_{water} was multiplied by 50 in all three spectral ranges. We used a 24 bit color coding so that the red, green, and blue channels each had 8 bits. In this case, the radiance value ranges between 0 and 255. If after brightening an I-value became overexposed, i.e., larger than 255, I was set to 255. As a result of this procedure, several points became overexposed in all three spectral ranges. These points are white in the color picture. From the brightened picture of the sky and its mirror image, we cut an annular sector above and below the horizon, respectively. These annular sectors were transformed to rectangles. The rectangle containing the water-reflected skylight was vertically mirrored and positioned next below the rectangle of the sky.

C. Photography and Painting

Figure 1(B) shows the geometry when an observer (photographer or painter) is viewing the water surface from height h with an angle of view $\theta_{\text{Brewster}} = \arctan(m = 1.33) = 53°$ relative to the vertical. Since $\tan(\theta_{\text{Brewster}}) = m$, the horizontal distance of the center of the BDP is $d = mh$ from the observer. Thus, for $h = 2m$ and 10 m, for example, $d = 2.66$ m and 13.3 m, respectively. Over the last decades, we have surveyed galleries and the Internet to look for photographs and paintings containing various water surfaces. We were interested only in those photos and paintings on the water surface of which a dark region occurs. We took photographs about the BDP without a

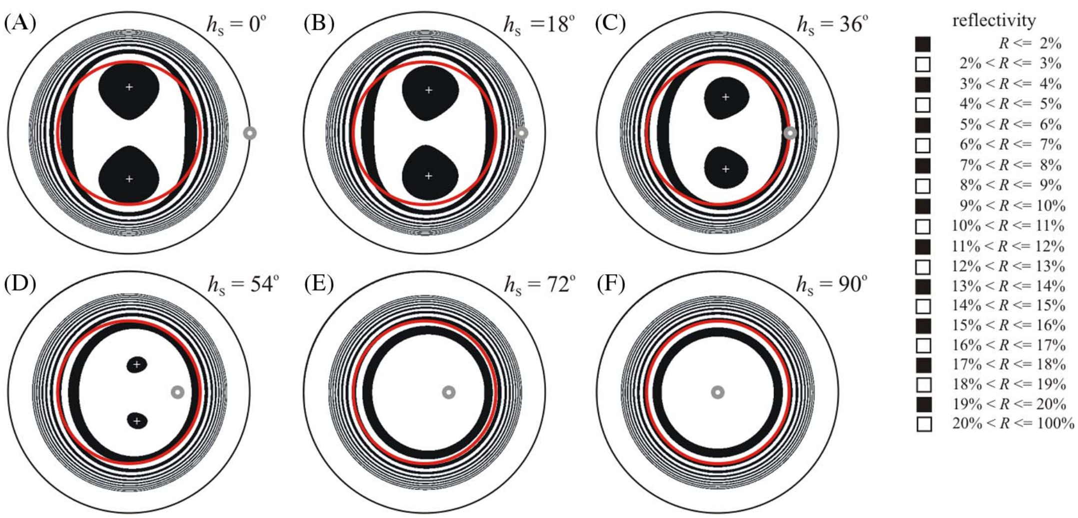

Fig. 2. Pattern of reflectivity R of the water surface calculated for clear skies with solar heights h_S = (A) 0°, (B) 18°, (C) 36°, (D) 54°, (E) 72°, and (F) 90°. The position of the mirror image of the sun is represented by a dot. The Brewster's point with minimum reflectivity R_{min} represented by + is considered as the center of the BDPs (the two central pear-shaped black patches, if they exist). In (A)–(D) the two central pear-shaped black patches represent regions with $R \leq 2\%$, corresponding with the BDPs, and the alternating white and black annular outer regions represent $2\% < R \leq 3\%, \ldots, 19\% < R \leq 20\%$, while the outermost white annular zone has $20\% < R \leq 100\%$.

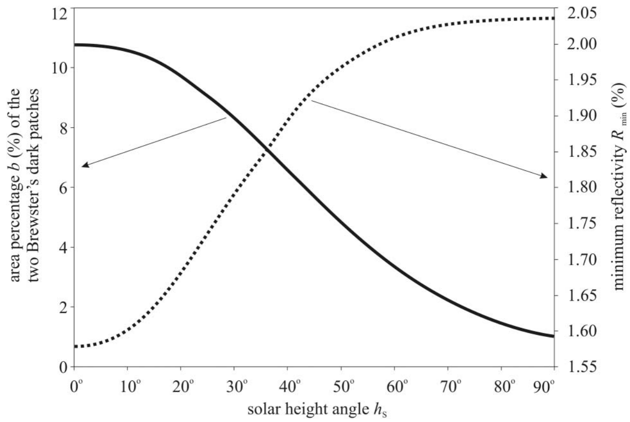

Fig. 3. Continuous curve with its vertical scale on the left side: Area percentage b of the two BDPs of the water surface relative to the whole downward pointing hemisphere filled by the water surface as a function of the solar height angle h_S computed for clear skies, the polarization patterns of which are described by the model of Berry *et al.* [6]. Dotted curve with its vertical scale on the right side: Minimum R_{min} of reflectivity of the water surface versus h_S computed for clear skies.

polarizer. We used a horizontally polarizing filter in front of the camera only when we wanted to demonstrate the vertical polarization of skylight along the solar vertical.

3. RESULTS

A. Brewster's Dark Patch

Figure 2 shows the pattern of reflectivity R of the water surface calculated for the clear sky with solar height angles $h_S = 0°$, 18°, 36°, 54°, 72°, and 90°. As h_S increases, the two BDPs of the water surface within the Brewster's annulus draw near to each other and are displaced to the solar half. At higher solar heights, the BDPs practically disappear [Figs. 2(D)–2(F)]. The BDPs ($R \leq 2\%$) have the largest area when the sun is on the horizon, their area decreases with increasing h_S, and they practically disappear if $h_S > 54°$ (Fig. 2). The centers of these dark patches, i.e., the Brewster's points with minimal R (represented by + in Fig. 2), approach each other with increasing h_S and shift toward the solar half of the water surface. The pear or egg shape of the BDPs is mirror symmetrical at $h_S = 0°$ and becomes asymmetrical with increasing h_S.

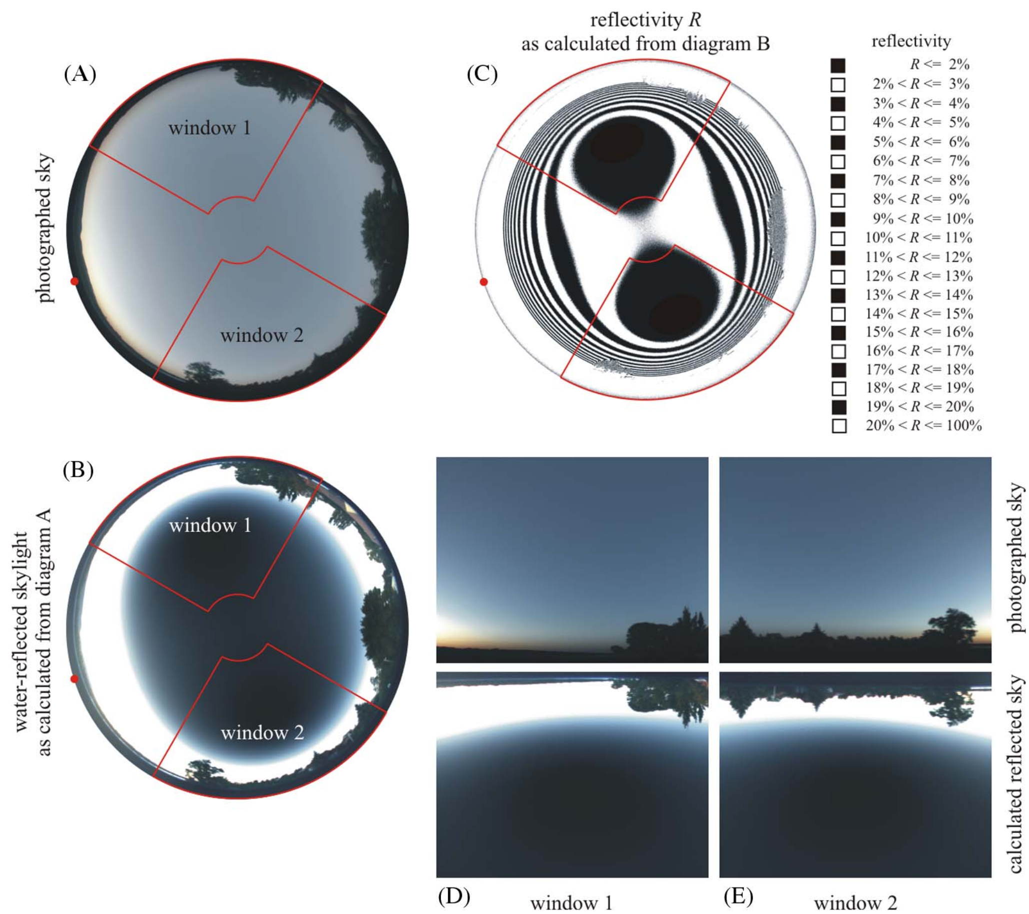

Fig. 4. (A) Color photograph of a clear sky at solar height $h_S = 0°$ taken with a 180° field-of-view fisheye lens. (B) Synthetic photograph of the mirror image of the sky in (A) with 50 times brightening, as computed from the polarimetric full-sky data simultaneously measured with picture (A). (C) Computed pattern of reflectivity R of the water surface under the sky in (A). The dark areas are the BDPs. In (A)–(C) the position of the (mirror) sun is marked with a red dot. (D), (E) The regions of the sky and the sky-mirroring water surface in windows 1 and 2 of (A) and (B), represented as rectangular photographs, showing the view an observer (painter) would see when looking perpendicular to the solar vertical. Their horizontal and vertical fields of view are 90° and 72°, respectively. The picture was taken by Gábor Horváth at Göd, Hungary (47°70′ N, 19° 15′ E) on 7 August 2012 at 20:15 (UT + 2h).

The continuous curve in Fig. 3 shows the area percentage b of the BDP relative to the water surface as a function of the solar height h_S computed for clear skies, the polarization patterns of which are described by the model of Berry *et al.* [6]. If $h_S = 0°$ (sun on the horizon at sunrise and sunset), b is 11.5%, it steeply decreases with increasing h_S, and it approximates 0% for $h_S > 58°$. From this, we conclude that the BDP exists practically only for solar heights $h_S < 58°$. The dotted curve in Fig. 3 represents the minimum R_{min} of reflectivity of the water surface versus h_S computed for clear skies. At $h_S = 0°$, the minimum reflectivity R_{min} is 1.58%, it steeply increases with increasing h_S, and, for $h_S > 60°$, it increases slowly to 2%.

B. Synthetic Photographs

In Figs. 4–6, the synthetic pictures of the radiance of the reflection on water are based on real measurements of the momentary state of polarization of the photographed sky and clouds performed with full-sky imaging polarimetry.

Figure 4(A) shows the color photograph of a clear sky at sunset with solar height $h_S = 0°$ taken with a 180° field-of-view fisheye lens. Figure 4(B) displays the computed mirror image of this sky with 50 times brightening. Figure 4(C) represents the computed pattern of reflectivity R of the water surface under this clear sky. Figures 4(D) and 4(E) show the view an observer can see when looking perpendicular to the solar–antisolar vertical at the sky and its water-reflected mirror image. The regions of the sky and the sky-mirroring water surface in windows 1 and 2 of Figs. 4(A) and 4(B) are in Figs. 4(D) and 4(E) transformed to rectangles, respectively. On the basis of Fig. 4(C), both BDPs fall into windows 1 and 2, therefore, they are clearly visible at the bottom of Figs. 4(D) and 4(E). A realistic painter should paint a similar view if she/he wants to display the spectacle of a water surface mirroring the sky and the vegetation on the horizon seen toward the north or south quarter perpendicular to the solar–antisolar vertical at sunset or sunrise.

Figure 5 shows a partly cloudy sky and its water-reflected mirror image at solar height $h_S = 26°$. According to Fig. 5(C), both BDPs are within windows 1 and 2. However, the dark patch is visible only in window 2 of Fig. 5(E), because window 1 is full of cirrus clouds with very low (<8%) degrees of polarization, so that the approximately unpolarized cloudlight is also effectively reflected at and near the Brewster's angle. Figure 5 also demonstrates well that the BDP preferentially occurs under a cloudless, clear sky, and very weakly polarized or unpolarized objects (here clouds) are mirrored in the patch's area, resulting in large brightness and color contrasts between the dark patch and the bright mirror image of unpolarized objects. We would like to emphasize, however, that clouds could indeed induce some polarization of the light within their own structure, and this is why the imaging polarimetry of the PARASOL/POLDER satellite sensor has been used to characterize the clouds' optical properties [8,9]. The radiation going through

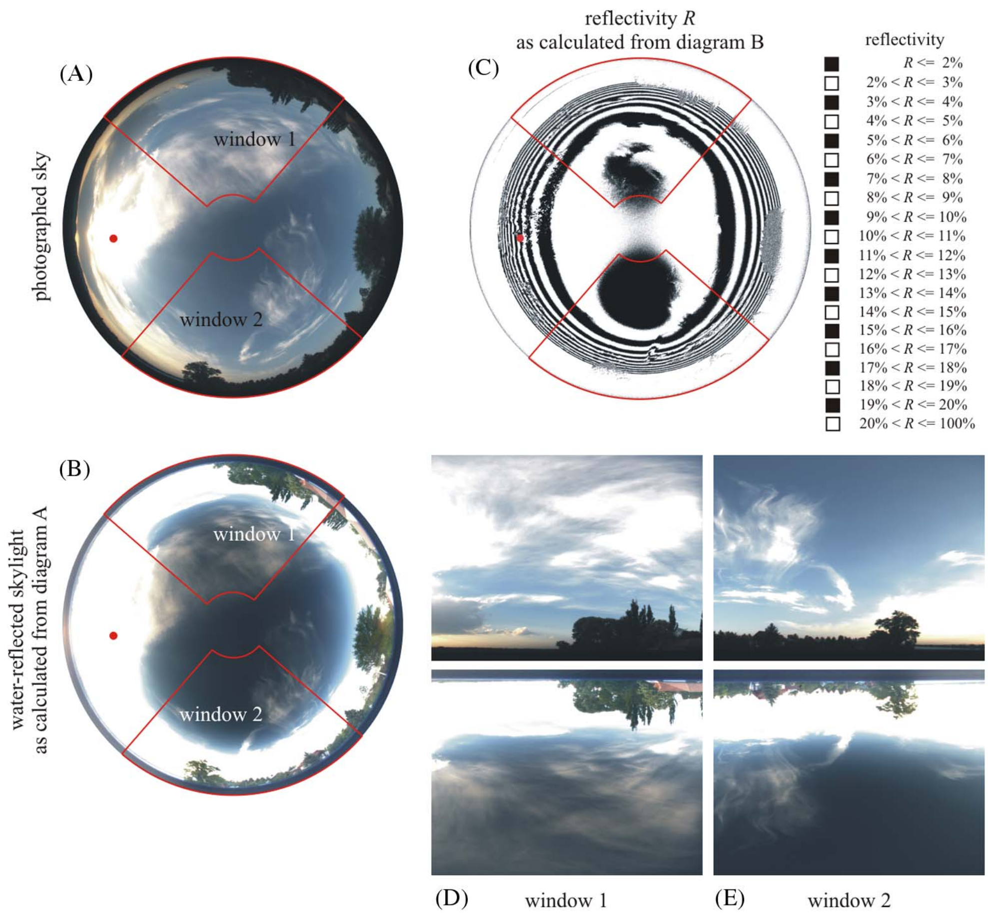

Fig. 5. As in Fig. 4 for a partly cloudy sky at solar height $h_S = 26°$ on 1 May 2012 at 16:38 (UT + 2h). The dark areas are the BDPs.

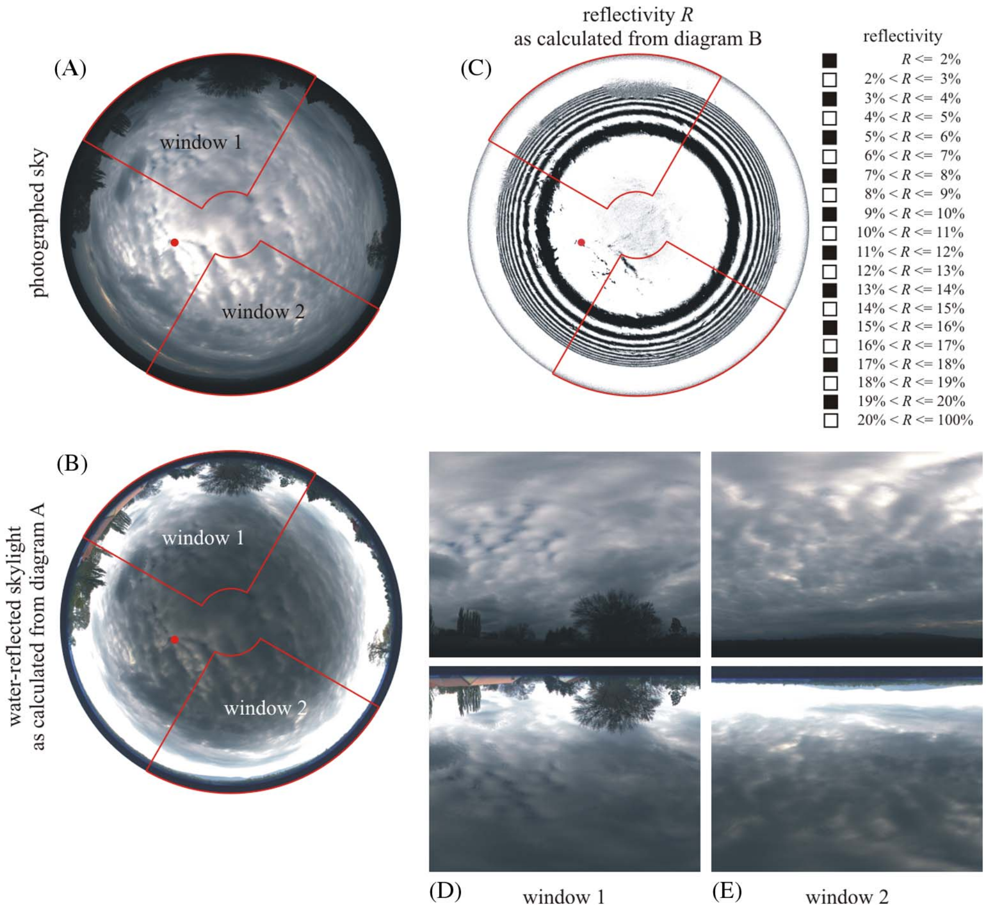

Fig. 6. As in Fig. 4 for an overcast sky at solar height $h_S = 58°$ on 15 April 2012 at 14:30 (UT + 2h). The BDPs do not show up in this diagram.

overcast skies and reaching the bottom of atmosphere (in this work the water surface) is often made isotropic due to the strong depolarization induced by the numerous multiple scattering events that occur inside the clouds.

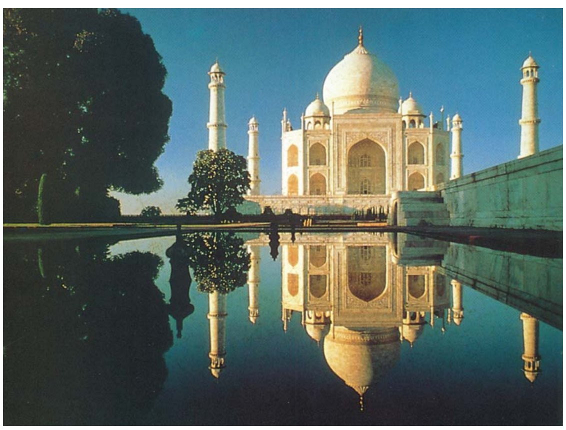

Fig. 7. Taj Mahal and reflecting pool looking to north at sunset. The nearer water does not reflect the blue sky, although it reflects the building very clearly. This proves that the darkening could not have been produced by a polarizing filter on the camera. This photograph originates from promotional material produced in 1995 by the travel company Abercrombie & Kent.

Figure 6 represents an overcast sky and its water-reflected mirror image at solar height $h_S = 58°$. The BDPs do not occur in Figs. 6(D) and 6(E) because, according to Fig. 6(C), such patches practically do not exist due to the very low (<6%) degrees of cloudlight polarization.

C. Photographs with a Brewster's Dark Patch on the Water Surface

Figure 7 shows the Taj Mahal with a reflecting pool looking towards the north at sunset. The nearer water surface does not reflect the blue sky, but it reflects the building very clearly. We think this photograph is an especially significant image, because the unpolarizing or only weakly polarizing dome and minarets are reflected very strongly against the BDP, showing that it could not have been done with a polarizing filter, which would have suppressed all reflections in this area. Note that a vertical linear polarizer could not darken the partially vertically polarized sky behind the Taj Mahal, only a horizontal polarizer would do this.

The photo in Fig. 8 was taken without a polarizing filter and shows a lake when the observer was facing north after sunrise in the early morning under a clear sky. The BDP is clearly visible at the bottom of the picture. The brown rocks in the water are also clearly visible because the light originating from them is not disturbed by the water-reflected skylight. In the BDP, the vertically polarized skylight is not reflected, thus one can easily see the underwater world (here, the rocks).

Fig. 8. Black-and-white picture of the Lake Hyllsjon near Havnmantorp and Lessibo, southern Sweden. The horizontal field of view of the camera was facing north. This photograph was taken by David Pye early on the morning on 4 June 2011 without a polarizing filter.

Figure 9 shows 180° field-of-view photographs taken about the sky and five different lakes through a horizontal polarizer and without such a filter at Göd, Hungary (47° 70′ N, 19° 15′ E) on 15 July 2017 at 20:30 (UT + 2h) when the horizontal optical axis of the camera aimed toward the west, south, and north. In Figs. 9(B) and 9(D), there is no BDP, since the observer looked toward the west, while in Figs. 9(F), 9(H), and 9(J) a BDP can be well seen on the water surface. In Figs. 9(A) and 9(C), the sky at and near the solar vertical is bright, because the skylight from the solar vertical is always horizontally polarized, which light is transmitted (practically) without attenuation through the horizontally polarizing filter in front of the camera. In Figs. 9(E), 9(G), and 9(I), however, the sky along the solar vertical is dark, because the skylight from the meridian perpendicular to the solar vertical is always partially vertically polarized, which light is considerably attenuated by the horizontal polarizer in front of the camera. In Fig. 9(F), the vertically polarized sky is a bright whitish blue behind the only weakly polarizing green tree. On the other hand, in the mirror image seen at the BDP on the water surface of Fig. 9(F), the mirrored tree remains bright green (due to its weak polarization), while the sky is dark blue behind this tree (due to the strong vertical polarization of skylight). Thus, the situation and the tree in Fig. 9(F) correspond to that of the Taj Mahal image in Fig. 2. Figures 9(E), 9(G), and 9(I) demonstrate well the vertical polarization of the sky when one looks toward the south or north, the consequence of which is the BDP on the water surface [Figs. 9(E), 9(G), and 9(I)].

D. Painting with a Possible Brewster's Dark Patch on the Water Surface

Figure 10 represents the painting of Edouard Vuillard entitled "The Ferryman" (1897, Musee D'Orsay, Paris [10]). In this picture, the rising/setting sun shines from the left, and the ferryman looks towards the south/north. The sky is almost clear, and the water-reflected light originates from the vertically polarized sky zone. Then, the dark Brewster's patch is correctly displayed on the water surface, especially in the bottom right corner of the picture. The pose of the oarsman does suggest that the artist was sitting at the back of the boat. The boatman's arm and hand on the oar actually show that he is facing us: the back of his head is seen only because he is looking over his shoulder

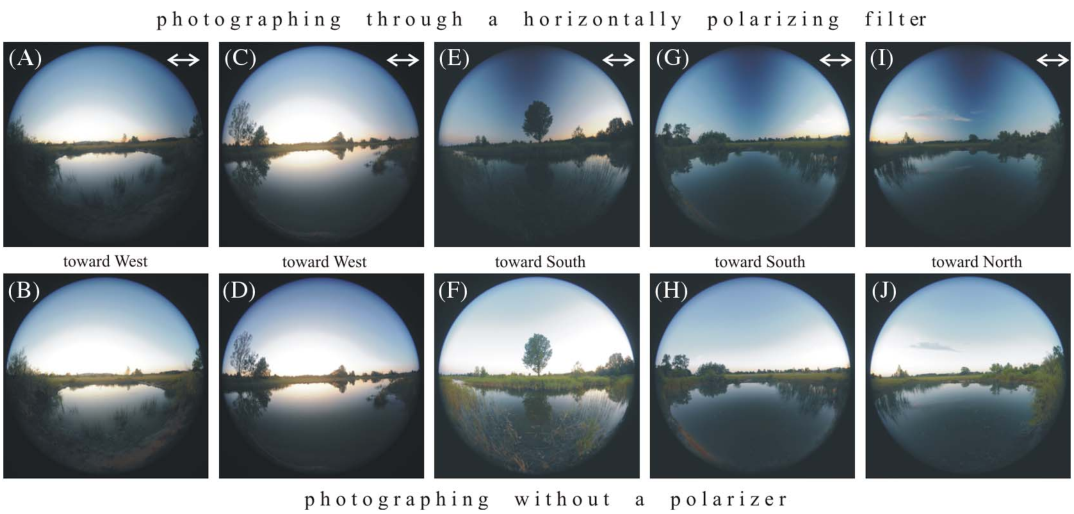

Fig. 9. 180° field-of-view photographs taken about the sky and five different lakes through a horizontal polarizer (A), (C), (E), (G), (I) and without such a filter (B), (D), (F), (H), (J) at Göd, Hungary (47° 70′ N, 19° 15′ E) on 15 July 2017 at 20:30 (UT + 2h) when the horizontal optical axis of the camera saw toward west (A)–(D), south (E)–(H), and north (I)–(J). In (B) and (D), there is no BDP, while in (F), (H), and (J) it can be seen on the water surface.

Fig. 10. Edouard Vuillard (1897): *The Ferryman* (Musee D'Orsay, Paris [10]). The dark patch at the water surface is most likely BDP. For detailed discussion, see text.

(to check where he is going, for example). This painting satisfies all the necessary criteria for the existence of BDP on the water surface. It is especially significant that the trees are reflected brightly, while the sky behind them is not.

4. DISCUSSION

The 180° field-of-view imaging polarimetry [3,4] made it possible to measure the reflection-polarization characteristics of the water surface under various sky conditions. With this technique, Gál *et al.* [5] measured the reflection-polarization patterns of a black water surface under a clear sky at sunset. Using the Mueller matrix of the air–water interface and the polarization patterns of the sky measured by full-sky imaging polarimetry, in this work, we computed the radiance (I) patterns of the mirror image of the sky seen on the water surface in the red, green, and blue spectral ranges. From these three I-patterns, we composed the synthetic color picture of the sky mirrored on the water surface in order to demonstrate the occurrence or non-occurrence of the BDP.

The polarization patterns of the clear (cloudless, fogless) sky are mirror symmetric with the symmetry axis coinciding with the solar–antisolar vertical. Due to this mirror symmetry, there are two BDPs positioned mirror symmetrically on both sides of the mirror solar–antisolar vertical (Fig. 2). If $h_S < 58°$, then the area of these patches is so small (Fig. 3) that they practically cannot be observed with the naked eye. The BDPs are largest and, thus, easiest to observe when the sun is on the horizon at sunset and sunrise (Figs. 2 and 3).

Können [1] wrote that when one sees this dark patch in rippled water, it is more like a triangle. Really, the shape of the computer-simulated BDP is triangular–ovoid, especially at lower solar heights (Figs. 2(A), 2(B), and 4(C)].

Although the angle of polarization pattern of totally overcast or foggy skies is very similar to that of the clear sky for the same sun's position [4,11], the degrees of polarization of such skies are usually so low that the BDPs practically cannot be observed with the naked eye even at low solar heights (Fig. 6). Figures 4(D), 4(E), 5(D), 5(E), 6(D), and 6(E) show the view that an observer can see when looking perpendicular to the solar–antisolar vertical at the sky and its water-reflected mirror image.

For obtaining the theoretical shape of the BDP, we used the model of Berry *et al.* [6] to describe the polarization patterns of the clear sky, which is a better description than the Rayleigh model. In the model of Berry *et al.* [6], the clear sky only contains air molecules (i.e., no aerosols). However, more or less aerosols are always present in the atmosphere. The impact of aerosols on our results is very weak for moderately turbid skies, since the degree of polarization p of skylight remains significant (i.e., not null). In the case of highly turbid atmospheres, such as skies dominated by desert dusts, coastal air pollution, or forest fire smoke [4,12], the p of skylight significantly decreases due to numerous multiple scattering events induced by the presence of dense aerosols. For such highly turbid atmospheres, despite the sky not being cloudy/overcast, aerosol loading leads to similar effects on celestial p values as cloudy skies (i.e., almost unpolarized skylight) [11]. In these cases, the BDP does not occur, like under overcast skies (Fig. 6).

In photos about waters, the majority of dark water regions can be explained simply by the use of a linearly polarizing filter in front of the camera's lens, presumably to enhance the visual appeal of the photograph: the transmission axis of the polarizer was vertical and, thus, blocked the horizontally polarized light reflected from the water surface near to Brewster's angle. In very rare cases, however, from different features of the photos, it can be guessed that such a polarizer was not applied during photography. When we ourselves took such photos, we did not use a polarizing filter.

In paintings, the dark regions on water surfaces can be trivially explained by the reflection of light originating from darker objects (e.g., buildings, trees, bushes) on the shore of the water body (e.g., pond, ditch, lake, river) or from distant dark clouds or mountains. We found only a very few paintings with a dark patch on the water surface, which cannot be explained in this way.

5. CONCLUSIONS

The BDP is seen at low solar heights (i.e., sunset or sunrise) near the Brewster's angle of 53° from the nadir on water surfaces when we look perpendicular to the solar vertical. Its existence is due to the fact that in the plane perpendicular to the solar vertical, the skylight incident onto the water surface at low solar heights is vertically polarized, thus leading to a negligible amount of water-reflected radiation when looking at the Brewster's angle. The BDP is rarely described in the literature, despite the fact that it is an easily observable, almost everyday occurring feature. Artists seldom illustrate the BDP because they generally paint back in the studio and forget exactly what was seen in nature.

In this work, we studied the radiance distribution of the BDP in still water as functions of the solar height h_S and sky conditions and obtained that the BDP exists practically only for $h_S < 58°$. We calculated the pattern of reflectivity R of a water surface for a clear sky and obtained from this idealized situation the shape of the BDP. From three full-sky polarimetric pictures taken about a clear, a partly cloudy, and an overcast sky, we determined the -pattern and composed from these synthetic color pictures showing how the radiance distribution of skylight reflected off the water surface, and how the BDPs would look under these sky conditions.

We presented photographs taken without a linearly polarizing filter about the BDP visible on the water surface when looking toward the north or south quarter perpendicular to the solar vertical at low solar heights near sunset or sunrise. We also presented the painting entitled "The Ferryman" of Edouard Vuillard (1897, Musee D'Orsay, Paris), on which a dark region on the surfaces of a river can be seen. We proposed that the dark patch of this painted water surface can be interpreted as an actual observation of the BDP.

Our results are of interest (i) to better understand the polarization optics of natural scenes showing the reflection of polarized skylight from water surfaces and (ii) to interpret pictures and paintings with regard to unexpected dark areas of the reflected skylight when the observation geometry corresponds to the plane perpendicular to the solar vertical.

Finally, we mention that it would be interesting to study whether fish-eating birds, for example, herons, hunting in shallow water at low solar heights orientate predominantly toward the north and/or south quarter, where due to the BDP they can look into water easier than in other directions of view, as has been suggested [2].

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