Underwater binocular imaging of aerial objects versus the position of eyes relative to the flat water surface

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The apparent position, size, and shape of aerial objects viewed binocularly from water change as a result of the refraction of light at the water surface. Earlier studies of the refraction-distorted structure of the aerial binocular visual field of underwater observers were restricted to either vertically or horizontally oriented eyes. Here we calculate the position of the binocular image point of an aerial object point viewed by two arbitrarily positioned underwater eyes when the water surface is flat. Assuming that binocular image fusion is performed by appropriate vergent eye movements to bring the object's image onto the foveae, the structure of the aerial binocular visual field is computed and visualized as a function of the relative positions of the eyes. We also analyze two erroneous representations of the underwater imaging of aerial objects that have occurred in the literature. It is demonstrated that the structure of the aerial binocular visual field of underwater observers distorted by refraction is more complex than has been thought previously.

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1. INTRODUCTION

The apparent position, size, and shape of an aerial object viewed by two eyes from under the water do not coincide with its true position, size, and shape, owing to the refraction of light at the water surface. According to Horváth and Varjú,[1] there are two possible apparent image positions of an aerial object point O viewed from under the flat water surface (Fig. 1): The first, C, is positioned where the line of the refracted ray entering the eye touches the evolute of refracted rays extrapolated backward, and the second, V, is at the point where the vertical line passing through O crosses the refracted ray extrapolated backward. Two common representations of this geometric optical problem are shown in Fig. 2. We show in this paper that these representations are erroneous.

The imaging of an aerial object point O viewed from water by a single eye is astigmatic because of refraction and the nonzero diameter of the pupil. The image of O is a slightly elongated vertical line at V (Fig. 1) with gradual blurring toward the ends, and the length of this image line depends on the shape of the pupil as well as on the distance of the eye from O. A single underwater eye must focus onto V to see O as sharply as possible.[1] With one eye alone, the human visual system is unable to determine the position of O in an unknown optical environment.[2,3] Thus all drawings of the apparent image position of aerial objects viewed from water by one eye alone are incorrect.

The apparent position of an aerial object viewed binocularly from water with a flat surface depends on the choice of refracted rays included in image formation[1]: If rays in a vertical plane containing the aerial object point O and both eyes E_1 and E_2 are considered (Fig. 3), the image point is at C, because lines e_1 and e_2 of the refracted rays entering the pupils intersect at C, while eyes E_1 and E_2 focus to V_1 and V_2, respectively. When rays emitted from O along a cone with a vertical axis through O are considered (Fig. 4), e_1 and e_2 cross at $V = V_1 = V_2$. Both eyes focus to V, where the binocular image is also perceived. All other rays from O do not intersect after refraction. This is the situation if the observer keeps its head obliquely with respect to the water surface, when e_1 and e_2 do not cross (Fig. 5). From this Horváth and Varjú[1] concluded that there is no binocular image formation at all for obliquely oriented eyes.

This is, however, not always valid. According to Fig. 5, looking at O with obliquely oriented eyes and focusing with E_1 to V_1 and with E_2 to V_2, the observer sees two distinct images V_1' and V_2' (not shown in Fig. 5) somewhere along the noncrossing lines e_1 and e_2, if the optical axes of the eyes do not coincide with e_1 and e_2. There are two points, K_1 and K_2, which are the nearest points from each other along e_1 and e_2, respectively, as shown in the inset of Fig. 5. The shortest section connecting e_1 and e_2 is K_1K_2, the bisecting point of which is K. Lines e_1 and e_2 converge in the plane that passes through K and the optical centers of eyes E_1 and E_2, while they diverge perpendicularly to this plane. If the viewing directions of the two eyes can converge and diverge appropriately in the plane through K and the optical centers of the eyes and perpendicularly to it, respectively, in such a way that the optical axes of the eyes coincide with e_1 and e_2, then the mentioned images V_1' and V_2' are fused into a binocular image positioned at K. In other words, binocular image fusion is performed at K by appropriate vergent eye movements if V_1' and V_2' are brought onto the fovea

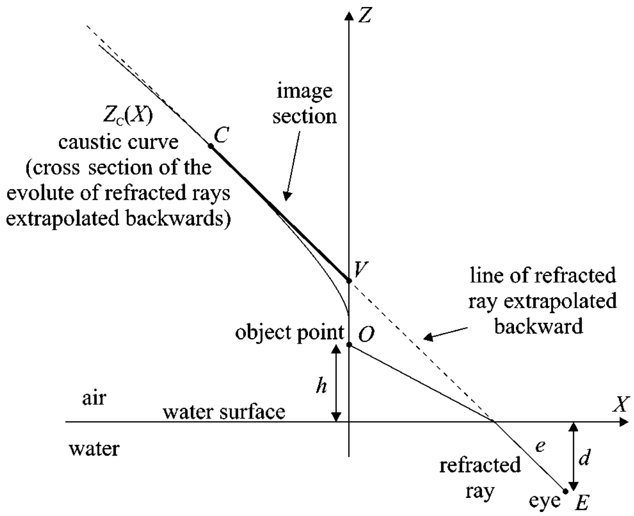

Fig. 1. Geometry of refraction of a ray of light originating from an aerial object point O and entering an underwater eye E when the water surface is flat. According to Horváth and Varjú,[1] C and V are the two possible apparent image points of O.

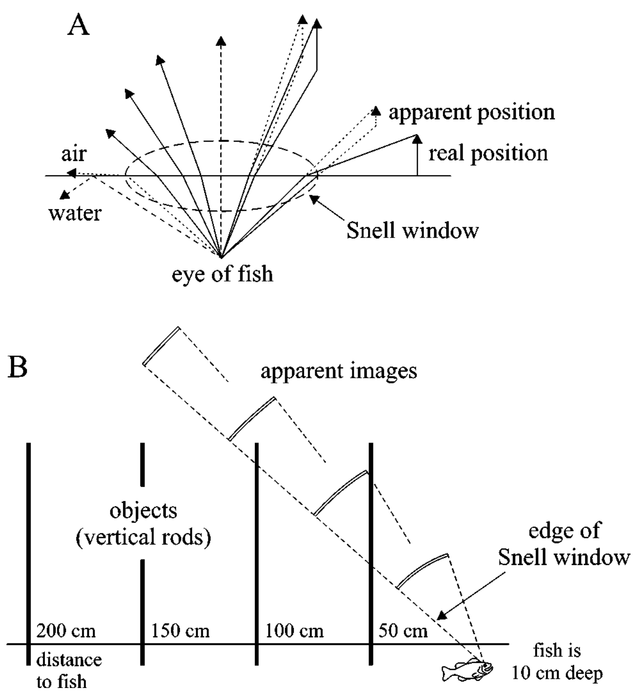

Fig. 2. Two erroneous representations of the apparent images of aerial objects viewed from water. The apparent positions of the aerial objects viewed by the underwater eye(s) are incorrectly drawn horizontally closer to the observer than the true horizontal distance. A is redrawn after Ref. 10, B after Ref. 11.

of eye E_1 and E_2, respectively. In humans, the optical axes passing through the foveae and the optical centers of the eyes can converge strongly in a plane through the optical centers but can diverge only slightly (a few degrees) perpendicularly to it.[3] Thus if the minimum distance K_1K_2 is too large, binocular fusion cannot be performed, and the observer sees two distinct images along e_1 and e_2 at an indefinite distance. Hence the smaller K_1K_2 is, the greater is the chance of the existence of the binocular image point K of the object point O. When eye E_2 rotates around eye E_1 in such a way that the baseline between them changes from vertical to horizontal, then K_1, K_2 move from C_1, C_2 to V_1, V_2 along the image sections, respectively, and K moves from C (Fig. 3) to $V = V_1 = V_2$ (Fig. 4).

In this paper we calculate the position of the binocular image point K for an aerial object point O viewed by two arbitrarily positioned underwater eyes when the water surface is flat. Assuming that binocular image fusion is performed by appropriate vergent eye movements, we compute and visualize the structure of the aerial binocular visual field determined by the binocular image points K as a function of the direction of the baseline between the eyes of an underwater observer for a flat water sur-

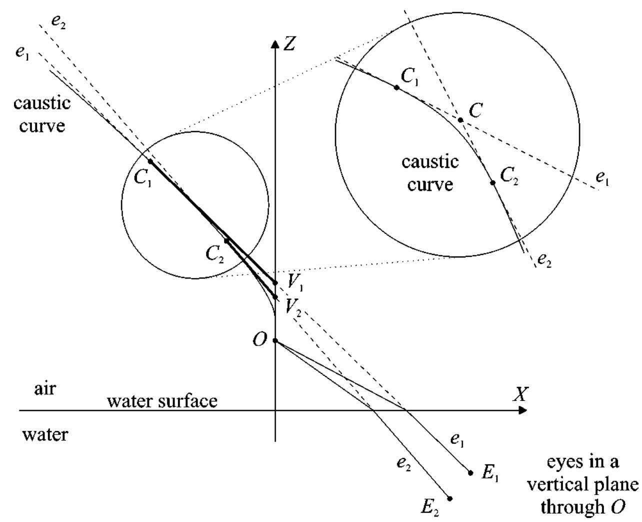

Fig. 3. If the eyes E_1 and E_2 of an underwater observer and the aerial object point O lie in the same vertical plane, the lines e_1 and e_2 of refracted rays extrapolated backward and entering the eyes intersect at point C. Thus C is the binocular image of O if the water surface is flat.

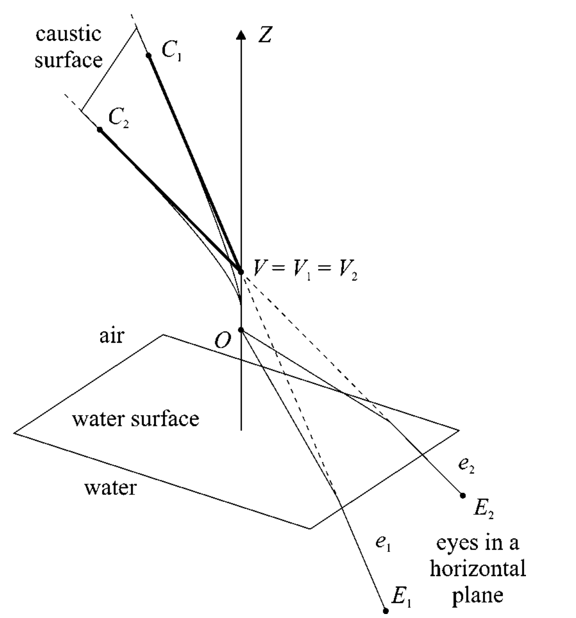

Fig. 4. When the two underwater eyes E_1 and E_2 lie in a horizontal plane, the refracted rays e_1 and e_2 extrapolated backward and entering the eyes intersect at point V. Thus V is the binocular image of O when the water surface is flat.

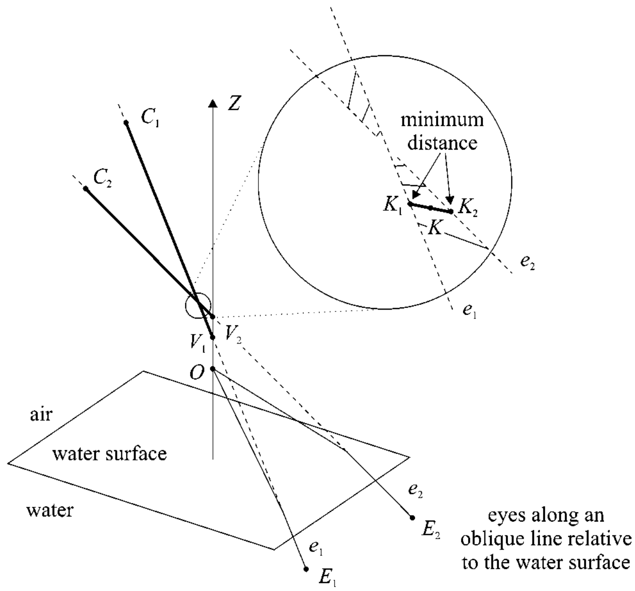

Fig. 5. When the two underwater eyes E_1 and E_2 lie along an oblique line relative to the flat water surface, the lines e_1 and e_2 of refracted rays extrapolated backward and entering the eyes do not intersect; they avoid each other in space. If the optical axes of the eyes coincide with e_1 and e_2 owing to appropriate vergent eye movements, the binocular image of O is K, which is the bisecting point of the shortest section K_1K_2 connecting the two nonintersecting lines e_1 and e_2.

face. The minimum distance K_1K_2 is also calculated as a function of the direction of view and of the degree of head tilting. Finally, two erroneous representations of the underwater imaging of aerial objects are analyzed. The reverse problem, the aerial binocular imaging of underwater objects versus the position of the eyes relative to the water surface was treated by Horváth and Varjú[4] and Horváth *et al.*[5] The present paper is a logical continuation of the work of Horváth *et al.*[5] and can be regarded as its second part.

2. MATERIALS AND METHODS

A. Caustic Associated with the Refracted Rays Extrapolated Backward

Since the topic of caustics in optics is a well-known subject,[6–9] here we deal only briefly with the caustic associated with the refracted rays extrapolated backward at the water–air interface. Consider an aerial object point O, from which rays of light start and propagate in different directions toward the flat water surface, where they are refracted. In this case the caustic is composed of two parts: (1) one is the evolute of refracted rays extrapolated backward and (2) the other is a segment of the vertical line through O. The evolute, called simply "caustic surface" in this paper, is a cylindrically symmetric surface, the rotation axis of which is the vertical line through O. Its vertical main cross section $Z_C(X)$ is called here simply "caustic curve" (see Fig. 1). Its expression was derived by Horváth and Varjú[1]:

$$Z_C(X) = hn[1 + (n^2 - 1)^{1/3}(x/hn)^{2/3}]^{3/2}, \qquad (1)$$

where $n = 1.33$ is the index of refraction of water and h is the height of O above the water surface. The line of a refracted ray extrapolated backward has two distinguishable points, C and V, at which it touches the caustic curve and intersects the vertical line through O, respectively. The straight line between C and V is called the "image section" (see Fig. 1).

B. Position of the Binocular Image Point of an Aerial Object Point for Arbitrary Positions of Underwater Eyes

Let the positions of the two underwater eyes be E_1 and E_2. The distance U between the eyes is constant and set as unit ($U = 1$). E_1 is fixed to axis Z at a depth $d = -2$ from the flat water surface, while the position of E_2 varies on the surface of a sphere, the radius of which is $U = 1$. This is the unity sphere of possible positions of E_2. The direction of the section connecting the eyes is characterized by angle θ measured from axis Z and by angle φ measured from axis X' in the plane of axes X' and Y' (Fig. 6B below). Our calculations are restricted to the positions of E_2 on the unity sphere characterized by $0° \leqslant \theta \leqslant 180°$ and $0° \leqslant \varphi \leqslant 90°$. The aerial binocular visual field for positions of E_2 outside this region can be obtained by appropriate rotation of the corresponding pattern calculated for a given position of E_2 within the region mentioned.

The coordinates of the nearest points K_1 and K_2 on lines e_1 and e_2 of the refracted rays entering the pupils were calculated by means of the same geometric optical method that was used in the work of Horváth *et al.*[5] The only difference is that now the eyes E_1 and E_2 are under the flat water surface (and not in air) and the object point O is in the air (and not under water). The binocular image point K of O is at the bisecting point of the section K_1K_2. The determination of the coordinates of K is approximate, since during the calculations an equation of fourth order had to be solved numerically for the angle of refraction of a refracted ray (e_1 or e_2) with the use of the tangent method of Newton combined with bisection.[5] The input data of these calculations are the coordinates X_O, Y_O, Z_O; X_{E_1}, Y_{E_1}, Z_{E_1}; X_{E_2}, Y_{E_2}, Z_{E_2} of O, E_1, and E_2. The computer program was developed by ourselves and written in program language C++ under Linux.

3. RESULTS

Figures 6–9 show how strongly the structure of the aerial world is distorted as a result of refraction at the flat water surface if viewed from the water binocularly as a function of angles θ and φ of eye E_2 on the unity sphere. A general feature is that the apparent height of all aerial points increases more or less: the greater the horizontal distance of an aerial point from the observer, the greater its apparent height. A cubic part of the aerial world is distorted and inflated in a characteristic height hyperboloid formation. Figures 6–9 demonstrate that the structure of the aerial world distorted by refraction depends strongly on the relative positions of the eyes.

Figure 6 shows the binocular image of an aerial vertical plane quadratic grid for several different positions of eye E_2 on the unity sphere. One can see that the binocular image of aerial vertical or horizontal lines suffers only a relatively small apparent distortion if the direction of view is near the vertical, and the image distortions increase as the viewing direction nears the edge of the Snell window. Depending on the position of E_2, the image of horizontal lines is generally a characteristic, nearly mirror-symmetric approximately hyperbolic curve (panels C, D, E, H, I, K, M, N). For certain eye positions, however, this shape becomes quite asymmetric (panels F, G, J, L). The images of vertical lines can be almost-vertical lines with smaller or larger bulges (panels D, F, G, H, I, J, L), or pipelike lines (panels C, E, K, M, N). We can see how the square-shaped cells of the aerial world are distorted to elongated or flattened deltoids or rhombi depending on the direction of view and the position of eye E_2.

Figure 7 demonstrates how a vertical section of the terrestrial world at the shore with a seal is distorted because of refraction if the observer looks out of the water with different positions of its eye E_2 relative to E_1. We can see that the shape of the seal is strongly elongated or bulged, depending on the position of E_2.

The left columns in Figs. 8 and 9 show the binocular image of an aerial horizontal and a vertical quadratic plane grid, respectively, as a function of the position of eye E_2. The characteristic, nearly hyperboloid surfaces in the left column of Fig. 8 demonstrate the apparent shape of the horizontal ceiling over a swimming pool that could

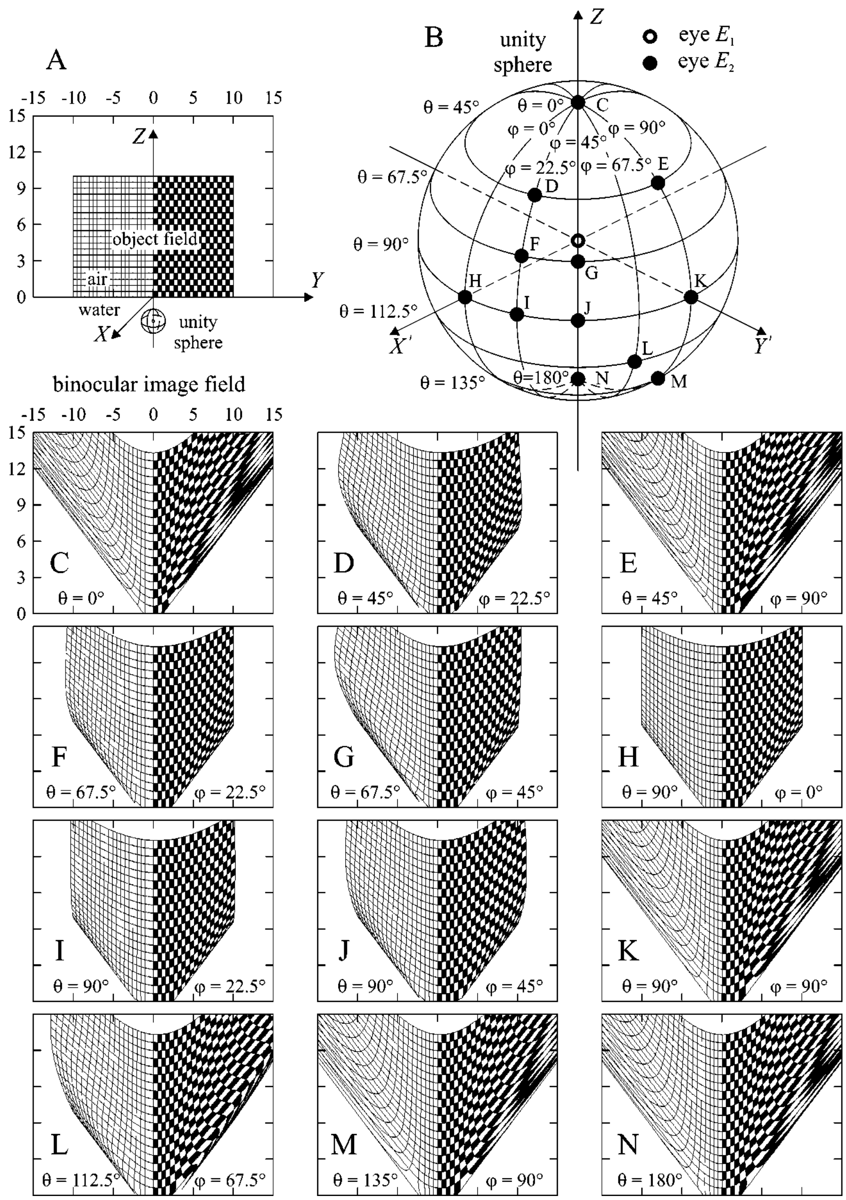

Fig. 6. Binocular imaging of aerial object points in a vertical plane as a function of the relative positions of the underwater eyes when the water surface is flat. A, An aerial vertical quadratic grid as object field, consisting of equidistant horizontal and vertical lines. For the sake of a better visualization, the cells of the grid are alternately painted white and black on the right half. The coordinates of the fixed underwater eye E_1 are $X = 0$, $Y = 0$, $Z = -2$. The small circle represents the unity sphere, at the center of which is E_1 and on the surface of which E_2 is situated. B, The positions of E_2 on the unity sphere for which the binocular images were computed. C–N, binocular images of the aerial grid in figure A as functions of angles θ and φ of E_2 on the unity sphere. In the calculations it was assumed that the binocular image point of every object point is the point K defined in Fig. 5.

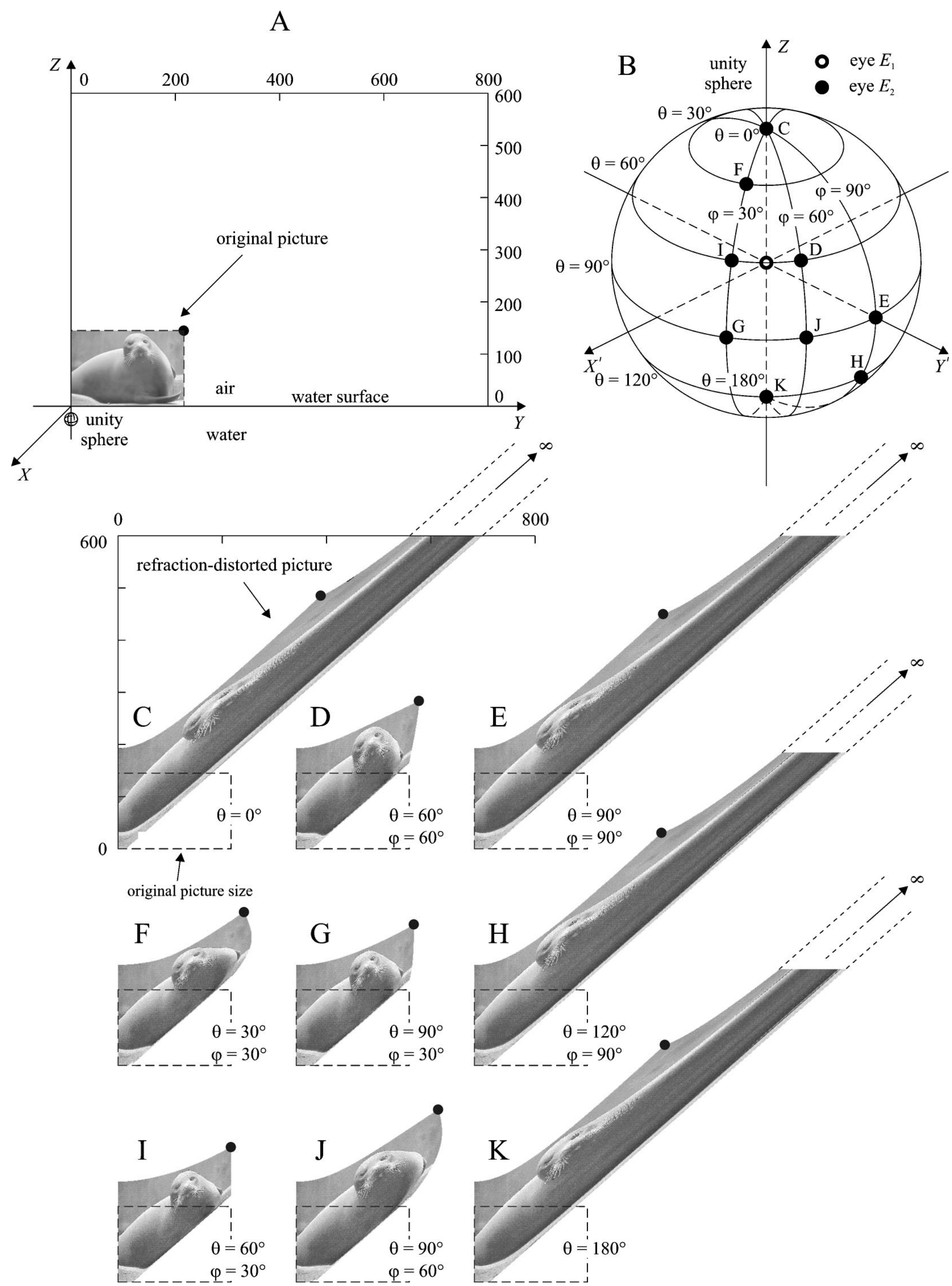

Fig. 7. As Fig. 6, but here the right half of the vertical grid is replaced by a picture representing a vertical section of the terrestrial world with a seal lying on the shore, and the calculation of the Y and Z coordinates of the binocular image point K was performed for every pixel (as object point) of this picture (pixel number $= 800 \times 600 = 480{,}000$).

be seen from the water as a function of the position of E_2. For a certain arrangement of eyes, the middle part of one side of the quadratic grid is distorted in such a way that the image becomes oar shaped (panel E). We can see in the left column of Fig. 9 that for certain eye positions (panels C–E) the binocular image of an aerial vertical grid is not a plane but is a slightly curved surface differing more or less from the plane of the aerial vertical object grid. At other positions of eye E_2, the binocular image remains exactly two dimensional, and the plane of the image is parallel to the original vertical grid (panels B, F).

The right columns in Figs. 8 and 9 show the minimum distance K_1K_2 between the two avoiding refracted rays of light e_1 and e_2 extrapolated backward and entering the eyes E_1 and E_2 as a function of the position of E_2. At the bisecting point of section K_1K_2 the binocular image point K of an aerial object point O is formed (Fig. 5) if binocular fusion is performed. The greater this minimum distance

K_1K_2, the larger the vergent eye movements needed for binocular fusion. In the two special cases of the eye positions studied earlier by Horváth and Varjú,[1] K_1K_2 is zero. Then the optical axes of the eyes need only to converge appropriately in a plane through the optical centers of the eyes. However, if K_1K_2 differs from zero, an appropriate divergence of the optical axes of the eyes perpendicularly to this plane is also necessary for binocular fusion.

4. DISCUSSION

In this work all calculations are performed for the case in which the depth d of the fixed eye E_1 below the flat water surface is $-2U$, where $U = 1$ is the eye distance set as unit (see Fig. 1). With increasing d, the dependence of the refraction-induced apparent distortion of the aerial world on the position of eye E_2 becomes weaker, but the gross structure of the aerial binocular image field remains qualitatively similar to the patterns in Figs. 6–9.

A. Analysis of Two Erroneous Representations of the Underwater Binocular Imaging of Aerial Objects

In the literature, some representations of the underwater imaging of aerial objects are incorrect. Two examples can be seen in Fig. 2 (see Refs. 10 and 11). In Fig. 2A only one eye is considered, and there is no information about the relative position of the second eye. However, without this information, this figure is necessarily incomplete, because the apparent position of the binocular image of an aerial object viewed by two underwater eyes depends strongly on the relative position of the second eye, as we have shown in this work. In both Figs. 2A and 2B the apparent image of aerial objects are correctly shifted

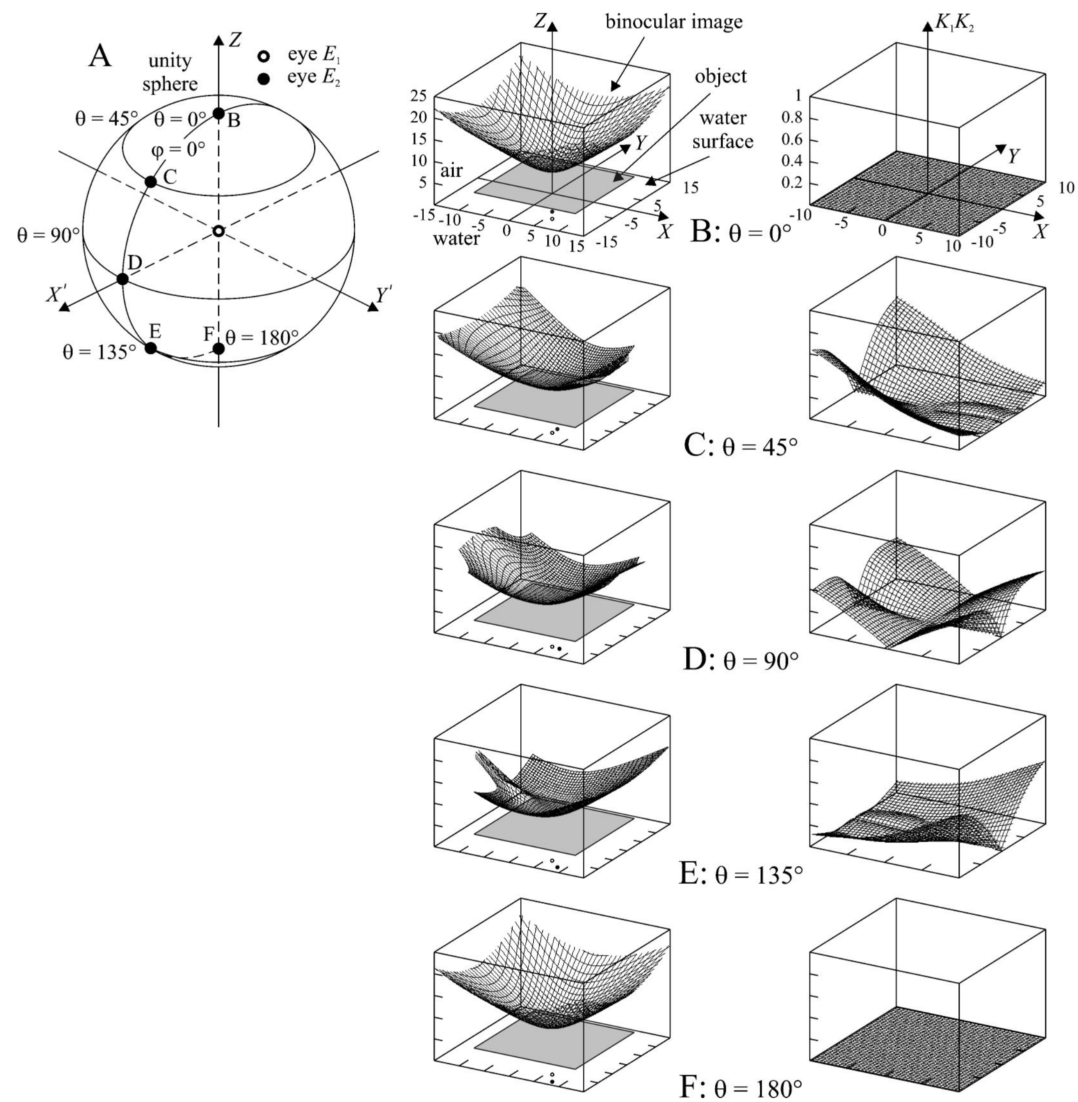

Fig. 8. Binocular imaging of aerial object points in a horizontal plane versus the relative position of the underwater eyes for a flat water surface. A: Eye positions for which computations were done. Left column in rows B–F: binocular image of the horizontal ceiling of a swimming pool (height $Z = 4$) viewed from the water through the flat water surface ($Z = 0$) as a function of the angle θ of eye E_2 with respect to eye E_1 for $\varphi = 0°$. The positions of the eyes are shown by dots. In the calculations it was assumed that the binocular image point of every object point is the point K defined in Fig. 5. Right column in rows B–F: the distance K_1K_2 between the two nearest points K_1 and K_2 of lines e_1 and e_2 of the refracted rays entering the eyes (Fig. 5) as functions of X and Y in three-dimensional perspectivic representation.

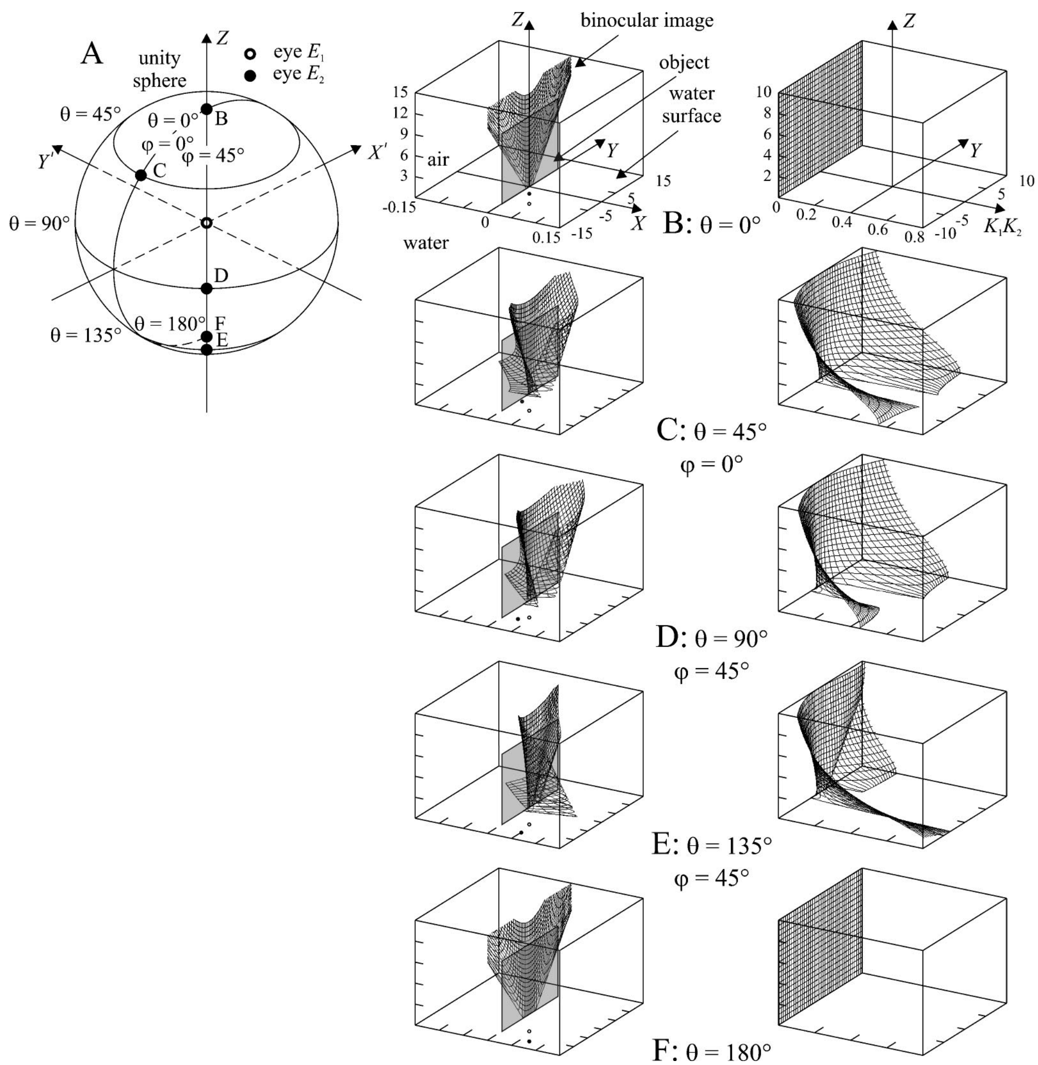

Fig. 9. As Fig. 8, but here the object is an aerial vertical quadratic grid (gray rectangle) positioned in the plane of axes *Y* and *Z*.

away from the flat water surface, but horizontally they are displaced incorrectly closer to the eyes, which displacement can never occur independently of the relative positions of the eyes. Irrespectively of the viewing method, the image is always shifted vertically further away from the flat water surface, but depending on the method of viewing, it shifts horizontally farther away from the observer (eyes positioned vertically or obliquely) or does not shift (eyes lying horizontally).

B. Fish That Have To Take into Consideration the Refraction during Prey Capture

The archerfish spits droplets of water at aerial insects, knocking them onto the water. Since the eyes of the fish remain below the water surface during sighting and spitting, the animal has to deal with potentially severe refraction effects at the water surface. The ability to cope with refraction has been demonstrated in two species of archerfish: *Toxotes jaculatrix* and *Toxotes chatareus* can correctly set their spitting angle to compensate for refraction.[12–14] They can correct for large refraction effects on the prey's apparent elevation or height. However, spitting accuracy decreases with increasing height or distance of the prey. These fish also correct for the curvature of the trajectory of water droplets. Since spitting velocity is relatively constant, the fish makes this correction by means of its spitting angle. *Toxotes jaculatrix* can also predict the point where the dislodged prey will later hit the water surface.[15] Until now the mechanism of depth perception in archer fish has not been studied.

The spatial relations between the apparent and the true positions of aerial objects viewed by fish may be of importance also for fishermen.[11] A fly fisherman, who sees a fish confronts the optical problem of where to cast the fly. Harmon and Cline[16] have looked into the optics of fly fishing. They believe the cast does have to be fairly accurate; otherwise, the image seen by the fish might be too distorted by refraction. They say that if we are fishing with a fly and can see the fish, we should cast the fly as close to it as we can. If we can put the fly within the fish's Snell window, it may be recognizable as a fly. If the fly is outside the Snell window, the separation of images of the part below the surface and of the part above makes the fly look less like a fly. The compression of the image of the part above the surface may even make that part so small that it is lost in the clutter at the edge of the Snell window.

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