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Resistivity Increase in Thin Conducting Films Considering the Size Effect

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A simple geometrical model is given for the resistivity increase of a thin conducting film considering the size effect. The results of the model agree well with the results of the Fuchs-Sondheimer non-geometrical theory.

Es wird ein einfaches geometrisches Modell für den Widerstandsanstieg einer dünnen Leiterschicht unter Berücksichtigung des Größeneffekts angegeben. Die Ergebnisse des Modells stimmen gut mit den Ergebnissen der nicht-geometrischen Fuchs-Sondheimer-Theorie überein.

1. Introduction

Because of the size effect the mean free path of conduction electrons is shortened in very thin conducting films. If the thickness of the film is commensurable with the mean free path (m.f.p.) of the electrons in a bulk material, the electrical resistivity increases because of diffusive scattering and reflection of the electrons at the film surfaces.

The theory of the size effect is elaborated by Fuchs [1] for the free-electron model and a spherical Fermi surface. This theory is developed by Price [2] for ellipsoidal Fermi surfaces. Sondheimer [3] elaborated the Fuchs theory for the explanation of the galvanomagnetic effects.

On the basis of the Fuchs-Sondheimer theory some expressions can be derived for the electrical conduction of thin conducting films, which can be used well in practice [4]. These expressions agree well with the experimental results [5 to 8].

The size effect occurs practically for all very thin films because they have an insulating (oxide) layer or an adsorbed gas layer on their surface in general, and the scattering of the electrons at the film surfaces is partly or totally diffusive [5 to 8]. The influence of the adsorbed gas layer on the resistivity of the thin metal films is examined by Finzel et al. [15].

The size effect appears in the multifilamentary superconducting composites too in their normal state if the filaments are appropriately close to each other, and the normal resistivity of the filaments is much larger than the resistivity of the matrix [9 to 14]. The resistivity increase caused by the size effect for the longitudinal resistance of the multifilamentary superconducting wires, tapes in normal state, causes a surplus resistivity [9 to 12]. A model is given by Cavalloni et al. [13, 14] to calculate the influence of the size effect on these multifilamentary systems.

In the granular superconducting composites the size effect plays a greater role in the normal state than in the continuous filamentary superconducting composites because in the granular composites there are small grains or short filaments with

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random distribution [16]. In such percolative systems these filaments can be very close to each other, and so the resistivity of the very thin matrix layers between these very close filaments can hardly increase in the normal state. The normal resistivity nevertheless plays an important role in the stabilization of the superconducting state of multifilamentary wires [16].

The size effect occurs in every polycrystalline metal, too, because the resistivity of the grain boundaries is larger than the resistivity of the crystallite. The resistivity increase in this case is not significant, because the difference between the resistivity of the crystallite and the grain boundary usually is small [17 to 19].

In this work a simple geometrical model is devised to calculate the resistivity increase as a consequence of the size effect. We examine conducting films with parallel surfaces which have insulating layers or adsorbed gas layers on their surfaces. We suppose that the electrons are scattered totally diffusively on the boundary surfaces of the film. We compare the results of our model to the results of the Fuchs-Sondheimer non-geometrical theory.

2. Size Effect of a Thin Conducting Film between Insulating Layers

In the free-electron model the resistivity is

$$\varrho = m_e \bar{v}/(n e^2 \lambda) \,, \tag{1}$$

where m_e, e, n, $\bar{v}$, and λ are the mass, the charge, the volume concentration, the mean velocity, and the bulk mean free path of the electrons, respectively. If the conductor has a finite extension in an insulating medium, then in the proximity of its surface the electron m.f.p. decreases because of the diffusive electron scattering on the scattering centres of the surface, and so the resistivity increases. This effect is called the size effect by us.

Our geometrical model to calculate this resistivity increase is a conducting film between two parallel insulating layers with infinite resistivity. Because of the infinite resistivity of the insulating layers the conduction electrons cannot be scattered beyond the boundaries of the film. Imagine a sphere with radius λ around the electrons, named λ-sphere in this work. In general the electrons are scattered if they have reached this sphere surface. In a homogeneous, isotropic bulk material the electron m.f.p. is isotropic because of the infinite extension. However, in a very thin conducting film between the above-mentioned insulating layers the boundary surfaces can cut a spherical calotte out of the λ-sphere. So the m.f.p. becomes direction dependent, namely in parallel to the insulating layers it does not change, but perpendicularly it decreases.

The average of this direction dependent m.f.p. can be calculated. Such a λ-sphere has a cylindrical symmetry with respect to the axis T going through the examined electron and perpendicular to the insulating layers (Fig. 1), so it is enough to make an average for the main section.

Consider an electron at a distance t_1 and t_2 from the film surfaces. In this case the average of the mean free path is

$$\bar{\lambda} = [\int_0^{\alpha} \lambda \, \mathrm{d}\theta + \int_0^{\beta} \lambda \, \mathrm{d}\theta + \int_{\alpha}^{\pi/2} (t_1/\sin\theta) \, \mathrm{d}\theta + \int_{\beta}^{\pi/2} (t_2/\sin\theta) \, \mathrm{d}\theta] \, \pi^{-1} \tag{2}$$

taking into account the notations of Fig. 1. Performing the integration we get from (2)

$$\bar{\lambda} = [\lambda(\alpha + \beta) - t_1 \ln \operatorname{tg}(\alpha/2) - (D - t_1) \ln \operatorname{tg}(\beta/2)] \, \pi^{-1} \,, \tag{3}$$

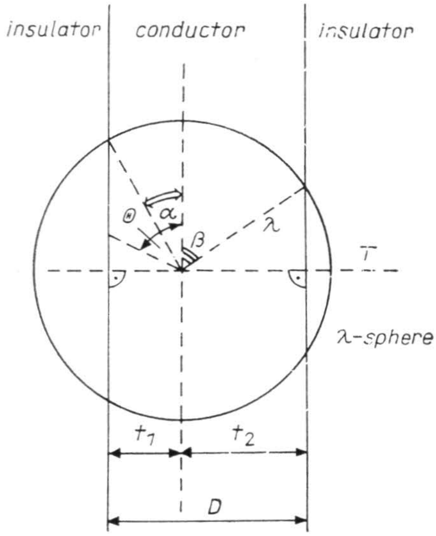

Fig. 1. The λ-sphere in a thin conducting film between two parallel insulating layers

where D is the thickness of the film. Let us introduce the following dimensionless quantities:

$$Q = D/(2\lambda)\,, \qquad x = t_1/\lambda\,, \qquad l = \bar{\lambda}/\lambda\,. \tag{4}$$

In the case of very thin films both the boundary surfaces cut the λ-sphere. In the case of thicker films such special arrangements exist when only one or none of the surfaces cuts the λ-sphere. Taking these arrangements into consideration the average of l depends on the quantities Q and x as follows:

1. if $Q > 1$ and
   - $0 \leqq x \leqq 1\,,$ then $l = [\alpha + \pi/2 - x \ln \operatorname{tg}(\alpha/2)]\,\pi^{-1} \equiv l_1$
   - $1 < x < 2Q - 1\,,$ then $l = 1$
   - $2Q - 1 \leqq x \leqq 2Q\,,$ then $l = [\beta + \pi/2 - (2Q - x) \ln \operatorname{tg}(\beta/2)]\,\pi^{-1} \equiv l_2$
2. if $1/2 \leqq Q \leqq 1$ and
   - $0 \leqq x \leqq 2Q - 1\,,$ then $l = l_1$
   - $2Q - 1 < x < 1\,,$ then $l = [\alpha + \beta - x \ln \operatorname{tg}(\alpha/2) - (2Q - x) \times$ $\times \ln \operatorname{tg}(\beta/2)]\,\pi^{-1} \equiv l_3$
   - $1 \leqq x \leqq 2Q\,,$ then $l = l_2$
3. if $0 \leqq Q \leqq 1/2\,,$ then $l = l_3$ (5)

If the conductor surrounded by the insulator with infinite resistivity has no parallel boundaries, we must average λ for the residual λ-sphere without the part(s) cut off by the boundary surface(s).

The functions $l(x, Q)$ and $l^{-1}(x, Q)$ are plotted in Fig. 2a and b. As we can see at larger values of Q the average of the electron m.f.p. decreases only in the proximity of the surfaces. The onset of the decrease is at the distance λ from the insulating layers. For distances larger than λ the average is equal to the bulk m.f.p. Accordingly for large Q the average resistivity increases in the proximity of the surfaces and for small Q the increase is extended over the whole film.

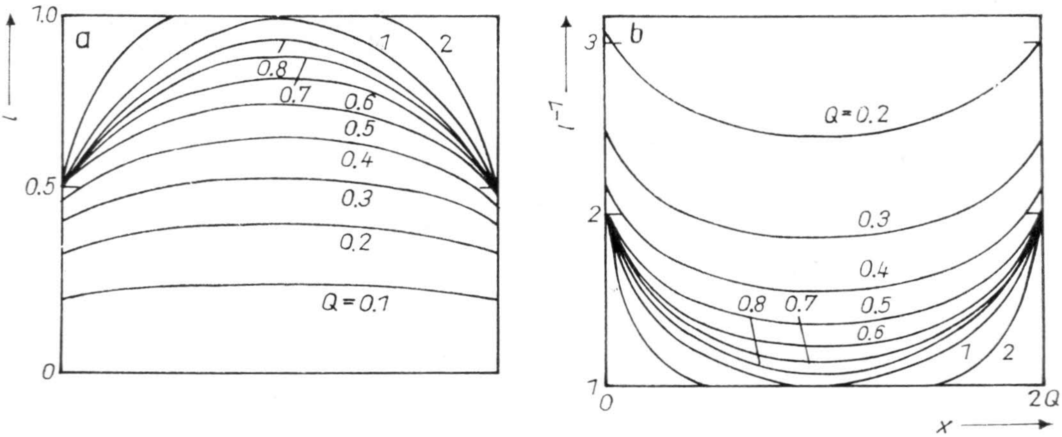

Fig. 2. a) The dependence of the dimensionless mean free path $l = \bar{\lambda}/\lambda$ on $x = t_1/\lambda$ and $Q = D/(2\lambda)$. b) The dependence of the dimensionless resistivity l^{-1} on x and Q

The average resistivity of the film is

$$\bar{\varrho}^{-1} = Q^{-1} \int_0^Q [\varrho(x)]^{-1}\, dx\,. \tag{6}$$

Using (1) and (6) we get

$$\bar{\varrho}^{-1} = (\varrho Q)^{-1} \int_0^Q l(x)\, dx\,, \tag{7}$$

where the functions $\varrho(x)$, $l(x)$ are symmetrical. Using (4), (5), (7) we obtain the following expressions:

1. $0 \leqq Q \leqq 1/2$

$$\bar{\varrho}^{-1} = (\varrho Q\pi)^{-1} \int_0^Q \left\{ \text{arc sin}\, x + \text{arcsin}\,(2Q - x) - x \ln \text{tg} \left(\frac{\text{arc sin}\, x}{2} \right) - \right.$$

$$\left. - (2Q - x) \ln \text{tg} \left[\frac{\text{arc sin}\,(2Q - x)}{2} \right] \right\} dx \equiv$$

$$\equiv (\varrho Q\pi)^{-1} \int_0^Q F(x)\, dx.$$

2. $1/2 < Q < 1$ (8)

$$\bar{\varrho}^{-1} = (\varrho Q\pi)^{-1} \left\{ \int_0^{2Q-1} \left[\pi/2 + \text{arc sin}\, x - x \ln \text{tg} \left(\frac{\text{arc sin}\, x}{2} \right) \right] dx + \right.$$

$$\left. + \int_{2Q-1}^{Q} F(x)\, dx \right\} \equiv (\varrho Q\pi)^{-1} \left\{ \int_0^{2Q-1} G(x)\, dx + \int_{2Q-1}^{Q} F(x)\, dx \right\}.$$

3. $Q \geqq 1$

$$\bar{\varrho}^{-1} = (\varrho Q)^{-1} \left[\pi^{-1} \int_0^1 G(x)\, dx + \int_1^Q dx \right].$$

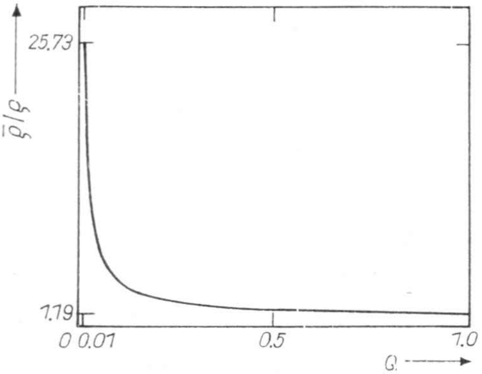

Fig. 3. The dependence of $\bar{\varrho}/\varrho$ on Q

Integrating (8), the average resistivities are

1. $0 < Q < 1/2$

$$\bar{\varrho}^{-1} = (\varrho Q\pi)^{-1}\left\{3(1-4Q^2)^{1/2}/2 - (1-Q^2)^{1/2} - 1/2 + 2Q \arcsin(2Q) - \right.$$

$$\left. - 2Q^2 \ln\left[\frac{1-(1-4Q^2)^{1/2}}{2Q}\right]\right\}.$$

2 to 3. $1/2 \leqq Q$

$$\bar{\varrho}/\varrho = 1 + (2\pi Q - 1)^{-1}. \tag{9}$$

The calculated $\bar{\varrho}/\varrho$ is plotted against Q in Fig. 3.

On the basis of the Fuchs-Sondheimer non-geometrical theory, the resistivity ratio

$$\bar{\varrho}/\varrho \approx 1 + 3/(16Q) = 1 + 0.1875/Q\,; \qquad Q \gg 1\,, \tag{10}$$

$$\bar{\varrho}/\varrho \approx -4/(3Q \ln Q) = -1{,}3330/(Q \ln Q)\,; \qquad 0 < Q \ll 1$$

can be obtained [1 to 4] if the scattering of electrons is totally diffusive at the film surfaces. From (9) we get

$$\bar{\varrho}/\varrho \approx 1 + 1/(2\pi Q) = 1 + 0.1590/Q\,; \qquad Q \gg 1\,, \tag{11}$$

$$\bar{\varrho}/\varrho \approx -\pi/(2Q \ln Q) = -1.5710/(Q \ln Q)\,; \qquad 0 < Q \ll 1\,.$$

It can be seen from (11) that the results of our simple geometrical model agree well with the results of the Fuchs-Sondheimer non-geometrical theory.

3. Conclusions

The size effect occurs in a lot of important systems: in very thin conducting films, in continuous filamentary and granular superconducting composites, and in polycrystalline metals. This effect increases the resistivity more or less significantly in these systems. The resistivity increase is described well by the Fuchs-Sondheimer non-geometrical theory. The results of our simpler, geometrical model for the resistivity increase agrees well with the results of the Fuchs-Sondheimer theory.

References

[1] K. Fuchs, Proc. Cambridge Phil. Soc. **34**, 100 (1938).
[2] P. J. Price, IBM J. Res. Developm. **4**, 152 (1960).
[3] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).
[4] K. L. Chopra, Thin Films Phenomena, McGraw-Hill Book Co., New York 1969.
[5] R. Nossek, Z. Naturf. **16A**, 1162 (1961).
[6] K. L. Chopra, L. C. Bobb, and M. H. Francombe, J. appl. Phys. **34**, 1699 (1963).
[7] M. S. P. Lucas, Appl. Phys. Letters **4**, 73 (1964).
[8] K. L. Chopra and M. R. Randlett, J. appl. Phys. **38**, 3144 (1967).
[9] V. M. Drobin, E. I. Dyachkov, I. S. Khukhareva, V. G. Luppov, and A. Nichitiu, Cryogenics **22**, 115 (1982).
[10] D. P. Lazar, N. M. Vladimirova, V. M. Drobin, E. I. Dyachkov, I. S. Khukhareva, Cryogenics **26**, 152 (1986).
[11] A. E. White, M. Tinkham, and W. I. Skocpol, Phys. Rev. Letters **48**, 1752 (1982).
[12] F. Nakane, T. Uchiyama, S. Nakamura, S. Miyake, and D. Ito, Proc. Internat. Cryogenic Mater. Conf., Kobe/Japan 1982 (p. 203).
[13] C. Cavalloni and R. Monnier, Helv. phys. Acta **55**, 669 (1982).
[14] C. Cavalloni, K. Kwasnitza, and R. Monnier, Appl. Phys. Letters **42**, 734 (1983).
[15] H. U. Finzel, E. Schmiedel, and P. Wissmann, Appl. Phys. A **42**, 87 (1987).
[16] Ch. Lobb, Techn. Rep. **16** (1980).
[17] R. A. Brown, J. Phys. F **7**, 1477 (1977).
[18] F. Lormand, Phil. Mag. **B44**, 389 (1981).
[19] F. Lormand, J. Physique **43**, 283 (1982).

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