Imaging polarimetry of the rainbow

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Using imaging polarimetry, we measured the polarization patterns of a rainbow on the shore of the Finnish town of Oulu in July 2001. We present here high-resolution color-coded maps of the spatial distributions of the degree and angle of linear polarization of the rainbow in the red (650 ± 30 nm), green (550 ± 30 nm), and blue (450 ± 30 nm) ranges of the spectrum. The measured polarization characteristics of the investigated rainbow support earlier theoretical and computational results and are in accordance with previous qualitative observations. To our knowledge, this is the first imaging polarimetric study of rainbow polarization. © 2003 Optical Society of America

OCIS codes: 010.3920, 100.0100, 120.5410, 260.5430, 280.0280, 290.1310.

1. Introduction

The rainbow, a colored circular band visible at ~42° from the antisolar point if sunlight falls onto water droplets underneath clouds, is one of the most spectacular phenomena in nature. It is not mere chance that many artists drew inspiration from rainbows.[1] One of the peculiar characteristics of rainbows is that rainbow light is strongly polarized with the direction of the E-vector tangential to the bow, as discovered by the French physicist Jean Baptiste Biot in 1811 (Ref. 2).

The light-scattering properties of large water spheres have been studied in great depth to explain the intensity and coloration of the light in the rainbow.[3–6] The polarization characteristics of the rainbow have also been investigated theoretically. Assuming unpolarized incident light on water droplets of various sizes, Dave[7] computed the intensity I and degree p of linear polarization of the scattered light. He concluded that there is a strong oscillation of p between the primary and the secondary rainbows as well as in the areas of the supernumeraries of the primary rainbow. Khare and Nussenzveig[8] proposed a theory of the rainbow and compared it with the exact Mie solution. Their improvement was particularly remarkable for electric polarization. Können and de Boer[9] extended the Airy theory of the rainbow to polarized incident light with the E-vector perpendicular or parallel to the scattering plane, which is radial to the bow at any point along the rainbow. Nussenzveig[10] developed a complex angular momentum theory of the rainbow including parallel and perpendicular polarization components. Mobbs[11] gave a rainbow theory based on Huygens's principle and compared it with the complex angular momentum theory. He found a good agreement over a large range of scattering angles and size parameters for both the magnetic and electric polarizations. Using Mie theory for monodisperse water drops, Lynch and Schwartz[12] calculated the degree of linear polarizationofrainbowlightwithoutbackgroundcontribution. They concluded that the maximum polarization $p_{\max}$ (occurring at the peak brightness of the bows) of both primary ($p_{\max} < 90\%$) and secondary ($p_{\max} < 50\%$) bows varies with drop size. Wang and van de Hulst[13] compared the results of Mie computations with those of the Airy approximation. Lee[14] compared the differences in the perceptible color and luminance as well as in the angular positions of luminance extrema between the Mie and the Airy rainbow theories.

The rainbow is a relatively exceptional atmospheric phenomenon, the polarimetric study of which is made more difficult by the fact that its appearance cannot be predicted: It occurs by chance when generally there is no polarimeter at hand, and if there is a polarimeter, one may usually wait a long time for the possible occurrence of a rainbow. Because of the

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Received 16 January 2002; revised manuscript received 21 March 2002.

0003-6935/03/030399-07$15.00/0

difficulties in observing rainbows, experimental research on the rainbow's polarization characteristics is scarce. Nevertheless, some experimental rainbow polarization research has already been published. For example, using a polar nephelometer employing a monochromatic linearly polarized laser source, Sassen[15] performed angular scattering measurements with vertically and horizontally polarized incident light to examine the rainbow generation of pendant water drops, a type of artificial near-spherical, vertically elongated particles, which model the distorted shapes of larger (diameter, >0.3–1 mm) raindrops with circular cross section in the horizontal plane as they fall. He measured linear polarization ratios as a function of scattering angle and compared the experimental data with theoretical predictions. Können[16,17] published a pair of color photographs taken by A. B. Fraser of a rainbow viewed through a linearly polarizing filter: in the left and the right photographs the rainbow light was maximally transmitted and extinguished, respectively. In the 1991 Light and Color in the Open Air feature issue of *Applied Optics*[18] some rainbow photographs taken by A. B. Fraser through linearly polarizing filters were also presented. Lee[19] used these photographs to isolate the rainbow's intrinsic colors, exploiting the fact that rainbow light is highly linearly polarized compared with light from the background: The rainbow's perpendicular polarization component is defined as that seen through a linear polarizer when its transmission axis is perpendicular to the scattering plane determined by the Sun, a raindrop contributing to the bow, and the observer. At this polarizer orientation, the rainbow is the brightest. If the polarizer is rotated by 90°, the bow's much weaker parallel polarization component is practically invisible. Light from the backround (landscape and cloudy sky) is usually almost unpolarized. Thus an estimate of the rainbow's intrinsic colors (without background) can be obtained if the digitized image of a rainbow's perpendicular polarized component is colorimetrically subtracted pixel by pixel from its parallel polarized counterpart.

The polarization pattern of rainbows would be difficult to study by a point-source scanning polarimeter, because rainbows are spatially extended and exist for a short period. These patterns can be measured only with wide field-of-view imaging polarimetry. To our knowledge, imaging polarimetric investigations of rainbows have not been published until now. During one of our field trips we were able to record the spatial distribution of the polarization of a rainbow. Here we present the polarization patterns of this rainbow measured in the red, green, and blue ranges of the spectrum.

2. Materials and Methods

Our team was lucky enough to get over all the difficulties in measuring the polarization patterns of a rainbow. When on 18 July 2001 at 17:42 (local summer time, UTC + 3; solar elevation, 29° 41′) we waited for a boat at the ferry port of the Finnish town of Oulu (65°0′N, 25°26′E), a beautiful rainbow occurred above the sea surface to the northeast. Since our 180° imaging polarimeter was at hand, because we were going to perform full-sky polarimetric measurements on the island of Hailuoto, we were able to take two triplets of polarization pictures from this rainbow with different exposures. Unfortunately, because of great haste, the color reversal film was put incorrectly into the roll-film camera of the polarimeter, and therefore the film was blocked after the first six photographs. Fortunately, one of the two triplets was successful with an optimal aperture and exposure combination. Then, on the way to Budapest, where the evaluation procedure was to be completed, one of the present authors (Bernáth) unfortunately forgot the hand luggage containing the developed film at Vantaa Airport in Helsinki while changing planes. Fortunately, Jari Toivonen (international liaison officer of Finnair) found this luggage and kindly sent the film to Budapest. This single successful triplet of polarization pictures is the basis for the polarization patterns presented in this study.

The polarization characteristics of the rainbow were measured in the red (650 ± 30 nm), green (550 ± 30 nm), and blue (450 ± 30 nm) ranges of the spectrum by a one-lens, one-camera, 180° field-of-view, rotating-analyzer imaging polarimeter. The optical axis of the fish-eye lens was horizontal and pointed toward the antisolar part of the horizon. The polarimeter, its calibration, and the whole evaluation procedure are described in detail elsewhere.[20] Here we mention only that the polarimeter is composed of a Nikon F801 roll-film camera equipped with a Nikon-Nikkor fish-eye lens (*f*-number, 2.8; focal length, 8 mm; angle of view, 180°) including a built-in filter wheel with three neutral gray linearly polarizing filters (type name HNP'B, Polaroid Corporation) with three different polarization axes ($\chi = 0°, 45°, 90°$ from the radius of the wheel). As a detector we used a Fujichrome Sensia II 100 ASA color reversal film. The spatial distributions of the measured degree of linear polarization $p = (I_{max} - I_{min})/(I_{max} + I_{min})$—where I_{max} and I_{min} are the maximum and the minimum of radiance, respectively, transmitted through the polarizer—and angle of polarization α (angle of the major axis of the polarization ellipse measured from the radius of the circular picture taken by the 180° field-of-view fish-eye lens of the polarimeter) are presented here in the form of high-resolution, two-dimensional, color-coded maps.

After evaluation of the color slides, we experienced that the observed (background included) degree of polarization of the primary rainbow was ~20–25%, which is much less than what one would expect from theory for the intrinsic (background excluded) polarization of rainbow light (that is, for a rainbow seen against a hypothetic black background).[2,7–14] The explanation of this is that we did not measure the intrinsic polarization of rainbow light alone, but the net degree of polarization of the mixture of the rainbow light and the unpolarized or weakly polarized light from the cloudy sky background. According to

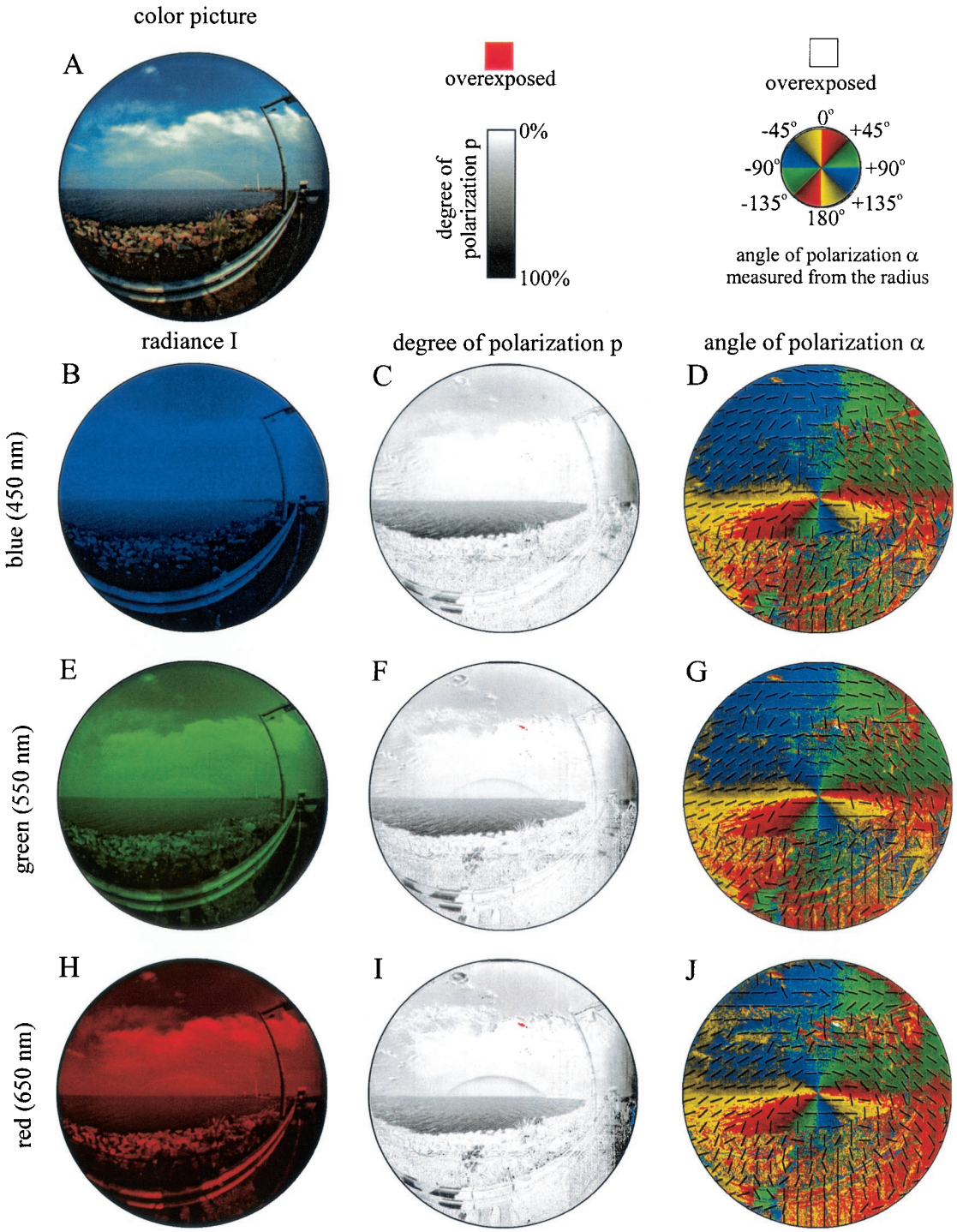

Fig. 1. 180° field-of-view color photograph of the shore of Oulu (65° 0′N, 25° 26′E, Finland) with a rainbow above the sea surface (A) and the patterns of radiance I (B, E, H), degree of linear polarization p (C, F, I) and angle of polarization α (D, G, J) measured by 180° imaging polarimetry on 18 July 2001, at 17:42 (local summer time, UTC + 3), in the blue (450 ± 30 nm), green (550 ± 30 nm), and red (650 ± 30 nm) spectral ranges, when the solar elevation was 29° 41′ above the horizon. Time of exposure, 1/250 s; aperture, 5.6; detector, Fujichrome Sensia II; 100 ASA color reversal film. At a given point of the circular patterns, α is measured clockwise from the radius. The black bars in the α patterns show the local direction of polarization at points of a quadratic grid.

Lee,[19] the light from the background desaturates and depolarizes the bow's intrinsic colors and polarization markedly. To estimate the degree of polarization of the rainbow light, we filtered the contribution of the background light in the following way: First we evaluated the raw polarization color pictures with the method described in Refs. 20 and 21. Then we determined an average radiance I_{av} of the sky background in the immediate vicinity of the primary rainbow, where I_{av} is the radiance if the polarizer's

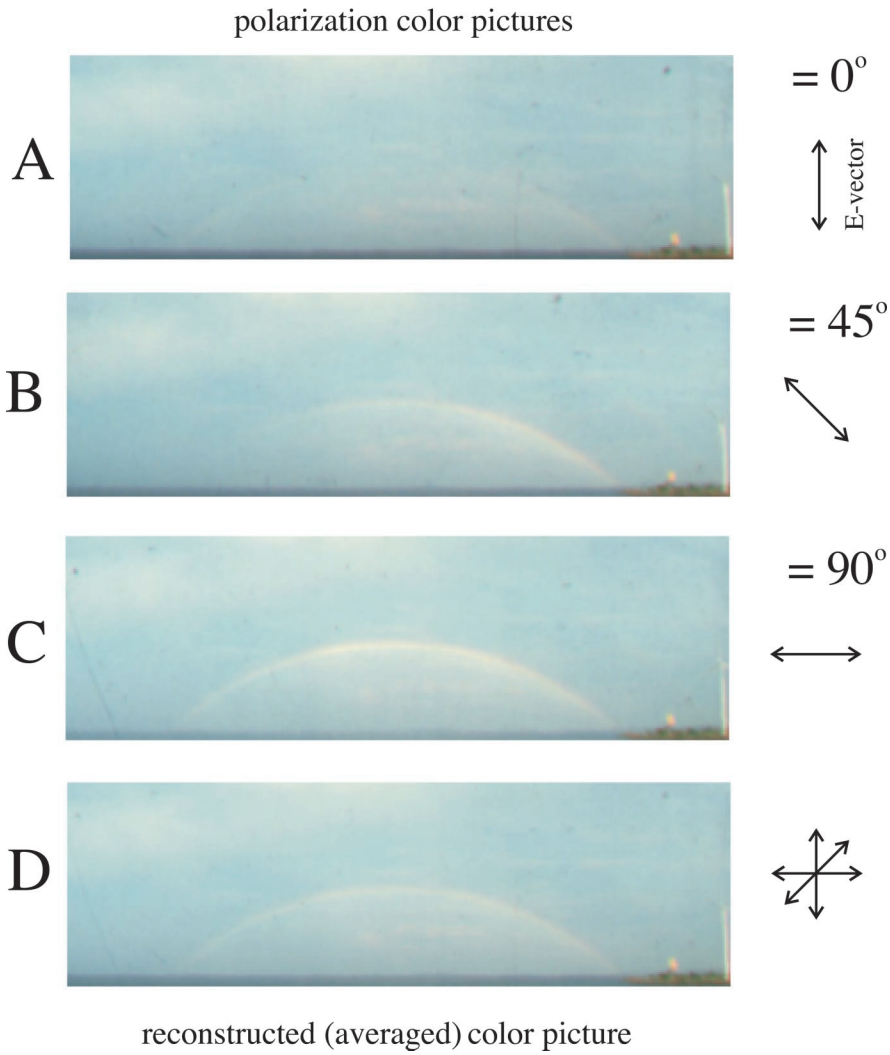

Fig. 2. (A–C) Enlarged rectangular parts of the three polarization color pictures of the rainbow in Fig. 1 taken through a linear polarizer with three different orientations χ (measured from the vertical) of the transmission axis (E-vector). (D) The averaged color picture (as could be seen by the human eye or could be photographed without a polarizer) computed from the three polarization pictures.

transmission axis is perpendicular to the E-vector direction in the investigated point. After subtraction of I_{av} from the measured radiance at each point of the picture, the usual evaluation process was performed again for the whole picture. The plots in Fig. 4 are obtained after this subtraction.

3. Results

Figure 1A shows the 180° field-of-view color picture (reconstructed from the three color photographs taken through a linear polarizer with three different transmission axes) at the shore of Oulu with a rainbow above the sea surface. In the foreground (bottom and right near the periphery) the barriers of an asphalt road, a reef of rocks, and the shadow of the observer and the polarimeter are visible. The sky above the observer is clear, but above the horizon the disappearing cumulonimbus clouds of a rainstorm are seen. The rainbow above the sea surface is produced by sunlight scattered by water droplets of this thunderstorm. Figures 1B–1J show the patterns of radiance I, degree of linear polarization p, and angle of polarization α measured at 450, 550, and 650 nm.

Figures 2A–2C show enlarged rectangular parts of the three polarization pictures of the rainbow. In Fig. 2D the reconstructed color picture of the rainbow is visible as can be seen by the human eye or can be photographed without a polarizer. Figures 2A–2C demonstrate how strongly polarized the sunlight reflected from the rain drops is and that the direction of polarization is parallel to the rainbow: As the polarizer rotates, parts of the rainbow are invisible, because the polarizer does not transmit the component of the electric field vector of the highly polarized rainbow light, which is perpendicular to its transmission axis. In Fig. 3 we see the rainbow in rectangular enlarged windows of the patterns in Fig. 1. The plots in Fig. 4 represent the p and α values measured at 450, 550, and 650 nm as a function of the angle of elevation θ along the vertical arrows in Fig. 3 after subtraction of the contribution of light from the sky background.

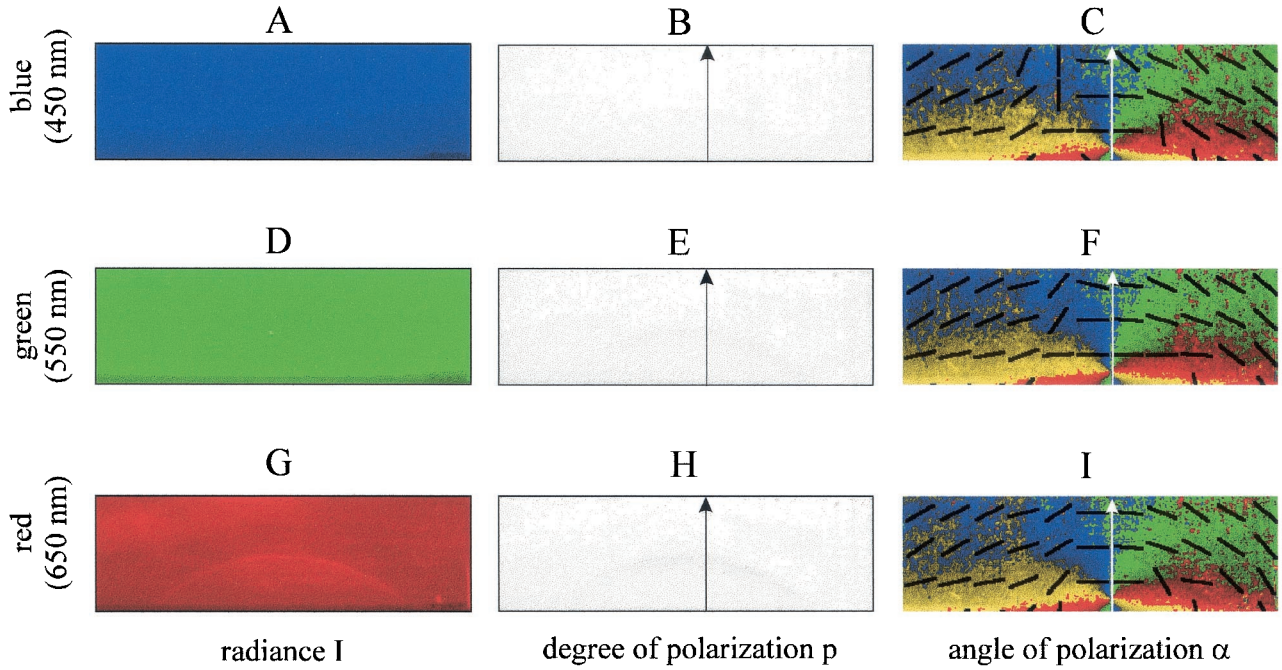

Fig. 3. Patterns of radiance I (A, D, G), degree of linear polarization p (B, E, H) and angle of polarization α (C, F, I) of the rainbow in Fig. 2 measured in the blue (450 nm), green (550 nm), and red (650 nm) spectral ranges. α measured clockwise from the radius and p are shown in the color wheel and gray-scale bar in Fig. 1, respectively. The black bars in the α patterns show the local direction of polarization. The plots in Fig. 4 are based on data measured along the vertical arrows B, C, E, F, H, I.

In the patterns of I (Figs. 1B, 1E, 1H; 3A, 3D, 3G) and p (Figs. 1C, 1F, 1I; 3B, 3E, 3H; 4A, 4C, 4E) one can see that the rainbow shows up most strikingly at 650 nm (red), whereas at 450 nm (blue) it is hardly visible. In the red part of the spectrum, in the p pattern (Figs. 1I, 3H, 4E) also the arc of the secondary bow is discernible. In the p patterns (Figs. 1C, 1F, 1I) we see that the light scattered by the clouds is almost unpolarized and that there is great contrast in p between them and the blue sky as well as the strongly polarized sea surface. At 650 nm the primary rainbow is as highly polarized as the water surface in the foreground. In the direction crossing the primary rainbow upward, there is an abrupt decrease of p: At 650 nm, for example, p decreases from ~50% of the primary rainbow to ~5% of Alexander's dark band between the primary and the secondary rainbows. The light in Alexander's dark band is unpolarized, because the background skylight is unpolarized. Lee[19] and Gedzelman[22] discussed the role of the background skylight in observations of the rainbow in the sky. In the direction downward from the primary rainbow, p gradually decreases with some oscillations (Figs. 1I, 3H, 4E). These oscillations are due to the supernumerary rainbows.

At all three wavelengths (450, 550, 650 nm), in the α patterns (Figs. 1D, 1G, 1J; 3C, 3F, 3I) and α plots (Figs. 4B, 4D, 4F) the rainbow does not show up, which demonstrates that there is no angle of polarization contrast between the rainbow and its celestial background. The situation is similar to the clouds: In the α patterns the clouds show up most strikingly in the red spectral range (Fig. 1J), but they are hardly visible in the blue part of the spectrum (Fig. 1D). The sea surface reflects highly and horizontally polarized light, independently of wavelength (Fig. 1).

4. Discussion

The primary rainbow observed by us, as is usual for rainbows, was red outside and blue inside, and the innermost colors were paler than the red. At ~11° outside the primary rainbow, a secondary rainbow with a reversed sequence of colors appeared, which was much fainter than the primary one. A few supernumerary rainbows were also visible below the primary bow. The investigated rainbow was seen, because the entire sky under the cumulonimbus cloud was filled with water droplets, and all of them were lit by the Sun.

According to Können and de Boer[9] as well as Können,[2] the strong polarization of the rainbow is the consequence of the path that the beams of light generating the rainbow follow in the drops: In the primary or secondary rainbow, the beams experience one or two reflections in the drop, respectively. Since these reflections happen at angles very near the Brewster angle, the reflected light is highly polarized, and the direction of polarization is always perpendicular to the scattering plane, which is radial to the arc of the bow. Since the light below the primary rainbow and above the secondary one rises in the same way as the light of the rainbow itself, its direction of polarization is also tangential to the bow. Though the scant light from the Alexander's band between the primary and the secondary rainbow arises from

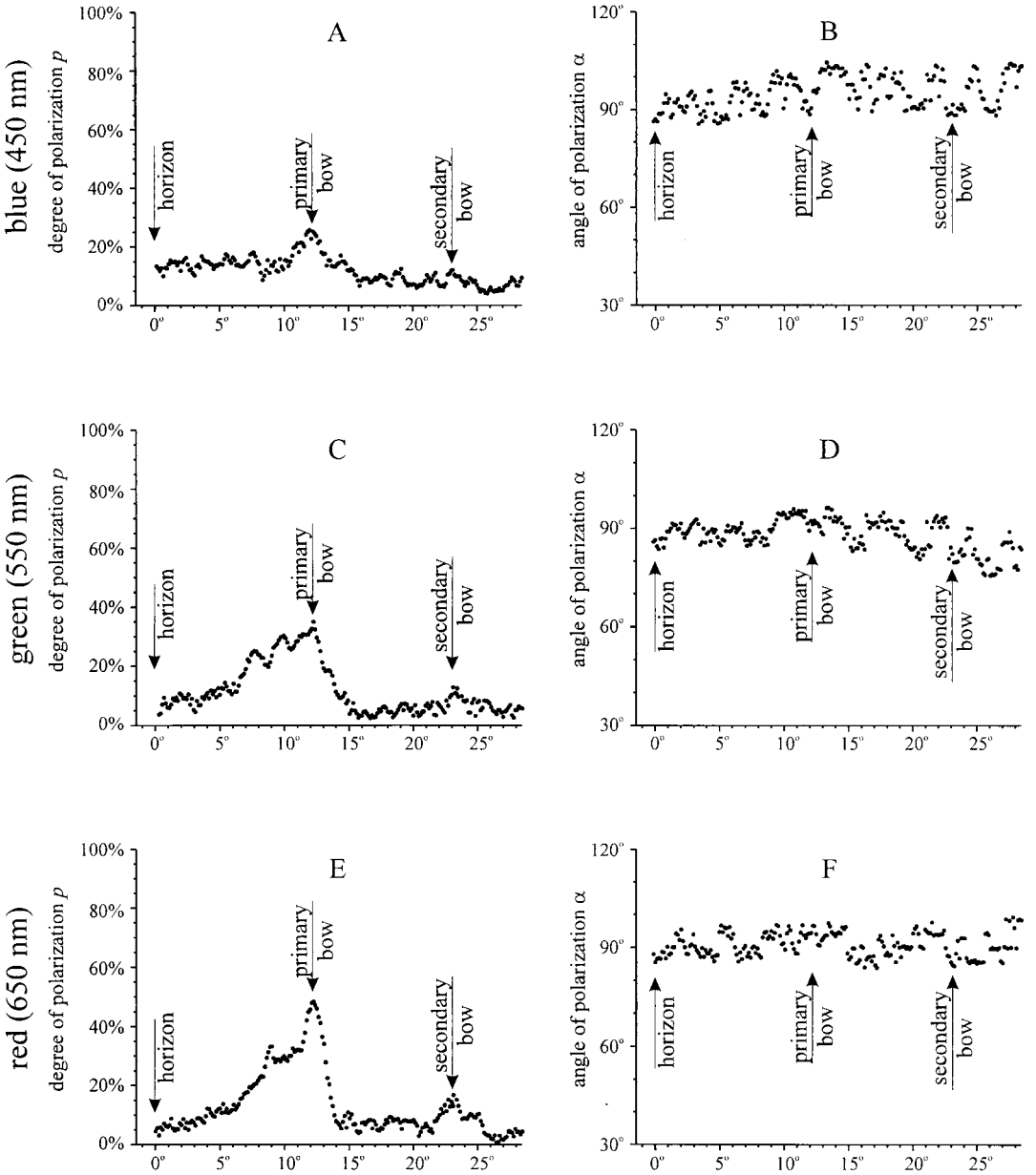

Fig. 4. Degree of linear polarization p (A, C, E) and angle of polarization α (B, D, F) in the blue (450 nm), green (550 nm), and red (650 nm) spectral ranges as a function of the angle of elevation θ (horizontal axis) measured along the vertical arrows shown in Fig. 3 (pointing from bottom $\theta = 0°$ to top $\theta = 28°$) after subtraction of the contribution of light from the sky background. α is measured clockwise from the radius of the original circular picture in Fig. 1.

reflections from the outside surface of the water droplets, background skylight plays a large role, too. This light is also tangentially polarized with respect to the bows.

Hence the direction of polarization of sunlight returned by the primary and the secondary rainbows, as well as by the celestial regions below the primary rainbow, between the primary and the secondary bows and above the secondary bow is always perpendicular to the scattering plane, that is, tangential to the bows. Thus there is no angle of polarization contrast between the rainbows and their sunlit celestial surroundings, which usually is also characterized by E-vectors perpendicular to the scattering plane. This effect is discussed in detail by Können.[2] This is why the rainbow does not show up in the α patterns and the α plots. The same phenomenon was recently observed by Pomozi *et al.*,[21] who demonstrated by full-sky imaging polarimetry that the clear-sky angle of polarization pattern continues underneath clouds if they and the underlying air layer are lit by direct sunlight. Under this illumination condition there is no angle of polarization contrast between the clouds and the blue sky (see also the α patterns in Fig. 1).

Our observation, that in the I and the p patterns

the investigated rainbow showed up best in the red part of the spectrum, but was hardly visible in the blue spectral range, can be explained by the logical assumption that the background light was unpolarized or only very weakly polarized as a result of multiple scattering, and it was most intensive in the blue spectral range because of Rayleigh scattering; therefore it could most strongly desaturate and depolarize the colored and polarized rainbow light in the blue part of the spectrum. Furthermore, the partially polarized light scattered in the air column between the observer and the rainbow, which is most intense in the blue, partly overwhelms the rainbow light, especially in the blue (see Figs. 4A, 4C, and 4E).

Gedzelman[22] developed a model for the brightness and coloration of rainbows that takes into account the cloud geometry and solar elevation with respect to the observer. The model consists of a beam of singly scattered sunlight that experiences depletion as it passes through the atmosphere and rainswath. It explains why the bottom of the rainbow tends to be both brighter and redder than the top when the Sun is near the horizon. Then the brightness of the bottom of the bow is affected most from scattering or absorption of light within the atmosphere. This is especially true for short wavelengths or for hazy conditions. One consequence is that the bottom is reddened in comparison with the top. The entire bow may be red when the Sun is at the horizon. Since our measurement was done at a relatively high solar elevation (29° 41′), this effect cannot explain why the rainbow that we investigated was most striking at 650 nm in both the I and the p patterns.

The optical phenomena associated with the rainbow are complex because of (i) diffraction of light by the raindrops, (ii) internal reflections within the drops, (iii) reflections from the outside surface of the drops, and (iv) interference of light. Further experimental research is needed to understand how these phenomena determine the polarization characteristics of the rainbow and to test the theoretical, computational predictions on rainbow polarization.

We conclude the following: Owing to several fortunate circumstances, we were able to perform what we believe is the first imaging polarimetric study of a rainbow. The patterns of the degree and angle of linear polarization of the rainbow were measured and visualized at 450, 550, and 650 nm. The measured polarization characteristics of the investigated rainbow support earlier theoretical and computational results and are in accordance with previous qualitative observations.

This research was supported by a Humboldt research fellowship from the German Alexander von Humboldt Foundation and by an István Széchenyi fellowship from the Hungarian Ministry of Education to G. Horváth. Further grants came from Oulu University and the Nowotka KG Bremen to V. B. Meyer-Rochow. We are grateful to R. Wehner (Institute of Zoology, University of Zurich, Switzerland) for loaning his Nikon-Nikkor fish-eye lens. Many thanks are due to Jari Toivonen (international liaison officer, Finnair), who found and kindly sent us our lost luggage with the polarization pictures of the rainbow presented in this study to Budapest. Many thanks are due to two reviewers for their valuable and constructive comments.

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