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Various sets of equations describe the geometric optics of the corneal lens of backswimmer, Notonecta glauca.

Notonecta glauca is an amphibian that always swims upside down under water, hence its name backswimmer. When waiting for its prey it hangs on the film of the water surface with its claws. However, if looking for a partner for copulation, or when the environment becomes unsuitable for living, it leaves the water and flies to another lake. Therefore, it is important that its optical apparatus works well in both media, water and air.¹

The backswimmer has a compound eye with ommatidia, each having a very flat corneal lens almost perpendicular to the axis of the ommatidium, with two parts separated by a bell shaped aspheric thin layer. In the aspheric layer the refractive index continuously changes from $n = 1.54$ to $n = 1.46$.

Most incident rays of light which are not parallel with the axis of the ommatidium are absorbed in the pigment cells around the cornea and the crystalline cone; thus they do not take part in image formation. As a result of the aspheric layer with continuously changing refracting index the spherical aberration is eliminated, and because of the flat external surface of the cornea the focus of the corneal lens does not change when the refractive index of the surrounding medium varies.

Our optical model assumes that the corneal lens and crystalline cone of the backswimmer have a cylindrical symmetry and that the aspheric layer can be replaced with an aspheric surface as shown in Fig. 1. (Only the half cross section of the cornea and crystalline cone is shown.) The shape of the external, aspherical, and internal surfaces of the corneal lens are described by the functions $f_1(x_1)$, $y(x)$, and $f_2(x_2)$, respectively, in the system of coordinates of Fig. 1. The shape of the corneal lens of the backswimmer is well described by the following equations²:

$$f_1(x_1) = 0, \quad f_2(x_2) = c(x_2/r)^2. \quad (1)$$

The definition of the geometrical parameters a, b, c, d, L , and

r of the cornea and crystalline cone can be read from Fig. 1. The refractive indices n_1, n_2, n_3, n_4 are the refractive index of the surrounding medium, the upper and the lower part of the cornea, and the crystalline cone, respectively.

The incident rays of light more or less parallel with the axis of the ommatidium cross after refractions of the same focal point F on the peak of the crystalline cone. Thus using the laws of refraction and some trigonometrical transformations the following sets of equations can be obtained (see Fig. 1):

$$x = x_1 - \tan(\alpha - \beta)[f_1(x_1) + a - y(x)], \quad (2)$$

$$x_2 = x - \tan(\omega - \delta + \alpha - \beta)[y(x) + b + c - f_2(x_2)], \quad (3)$$

$$L + f_2(x_2) = \frac{x_2}{\tan(\theta - \nu)}, \quad (4)$$

$$\tan(\alpha - \beta) = \frac{\frac{f_1'(x_1)n_1/n_2}{[1 + f_1'^2(x_1)]^{1/2}} - f_1'(x_1)}{1 - \frac{f_1'^2(x_1)(n_1/n_2)^2}{1 + f_1'^2(x_1)}}, \quad (5)$$

$$\frac{n_1/n_2}{[1 + f_1'^2(x_1)]^{1/2}} \frac{1 + f_1'^2(x_1)}{1 - \frac{f_1'^2(x_1)(n_1/n_2)^2}{1 + f_1'^2(x_1)}} = \frac{1}{\tan(\alpha - \beta)}$$

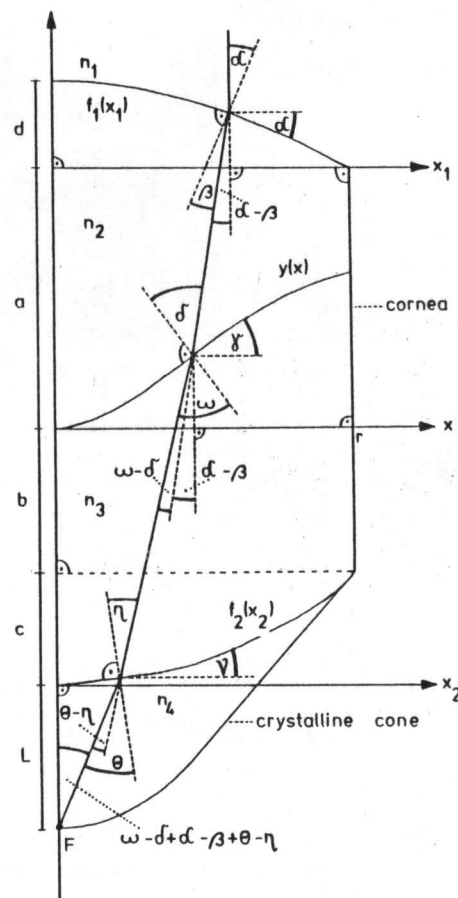


Fig. 1. Path of a ray of light parallel to the axis of the ommatidium in the corneal lens and the crystalline cone of the backswimmer.

$$\tan \delta = \frac{\tan(\alpha - \beta) + y'(x)}{1 - \tan(\alpha - \beta)y'(x)}, \quad (6)$$

$$\tan(\omega - \delta) = \frac{\frac{\tan \delta n_2/n_3}{[1 + \tan^2 \delta]^{1/2}} - \tan \delta}{1 + \frac{\frac{\tan^2 \delta n_2/n_3}{[1 + \tan^2 \delta]^{1/2}}}{1 - \frac{(\tan \delta n_2/n_3)^2}{1 + \tan^2 \delta}}}, \quad (7)$$

$$\tan(\theta - \nu) = \frac{\frac{\tan \eta n_3/n_4}{(1 + \tan^2 \eta)^{1/2}} - f'_2(x_2)}{1 + f'_2(x_2) \frac{\frac{\tan \eta n_3/n_4}{(1 + \tan^2 \eta)^{1/2}}}{1 - \frac{(\tan \eta n_3/n_4)^2}{1 + \tan^2 \eta}}}, \quad (8)$$

$$\tan \eta = \frac{\tan(\omega - \delta + \alpha - \beta) + f'_2(x_2)}{1 - \tan(\omega - \delta + \alpha - \beta)f'_2(x_2)}, \quad (9)$$

where $f'_1(x_1) \equiv df_1/dx_1$, $y'(x) \equiv dy/dx$, and $f'_2(x_2) \equiv df_2/dx_2$. Substituting Eq. (1) into this system of equations we obtain

$$P_{12}x_2^{12} + P_{10}x_2^{10} - P_9x_2^9 + P_8x_2^8 + P_7x_2^7 + P_6x_2^6 - P_5x_2^5 + P_4x_2^4 + \quad (10)$$

$$P_3x_2^3 + P_2x_2^2 - P_1x_2 - P_0 = 0,$$

$$y'^4 \left[1 - \frac{n_2^2}{n_3^2} (1 + z^2) \right] + 2zy'^3 + \quad (11)$$

$$y'^2 \left(1 - \frac{n_2^2}{n_3^2} \right) (1 + z^2) + 2zy' + z^2 = 0,$$

where the notations used in Eqs. (10) and (11) are the following:

$$k_1 = 1 + 4c^2L^2/r^4 + 4cL/r^2, \quad k_2 = 4c^4/r^8, \quad k_3 = 4c^2/r^4 + 8c^3L/r^6,$$

$$g_1 = (n_3L/n_4)^2, \quad g_2 = (n_3c/n_4r^2)^2 - (1 - n_3^2/n_4^2)k_3,$$

$$g_3 = 2cL(n_3/n_4r)^2 + (1 - n_3^2/n_4^2)k_1, \quad g_4 = (1 - n_3^2/n_4^2)k_2,$$

$$h_1 = 4cx(y + b + c)/r^2 - 2x, \quad h_2 = 1 + 4c^2(y + b + c)^2/r^4 - 4c(y + b + c)/r^2,$$

$$h_3 = 4c^2/r^4 - 8c^3(y + b + c)/r^6, \quad h_4 = 4c^4/r^8, \quad h_5 = 4c^2x/r^4, \quad Q_1 = 4c^2x/r^4,$$

$$Q_2 = (y + b + c)^2, \quad Q_3 = c^2/r^4, \quad Q_4 = 2c(y + b + c)/r^2 + 4c^2x^2/r^4,$$

$$Q_5 = 4cx(y + b + c)/r^2, \quad P_{12} = h_4g_4, \quad P_{10} = Q_3k_2 - h_4g_2 + h_3g_4, \quad P_9 = Q_1k_2 + h_5g_4,$$

$$P_8 = Q_3k_3 + Q_4k_2 - h_3g_2 + h_4g_3 + h_2g_4, \quad P_7 = h_5g_2 + h_1g_4 - Q_1k_3 - Q_5k_2,$$

$$P_6 = Q_4k_3 + Q_2k_2 + Q_3k_1 - h_2g_2 - h_4g_1 + h_3g_3 + x^2g_4,$$

$$P_5 = h_1g_2 + h_5g_3 + Q_5k_3 + Q_1k_1, \quad P_4 = Q_2k_3 + Q_4k_1 - h_3g_1 - x^2g_2 + h_2g_3,$$

$$P_3 = h_1g_3 + h_5g_1 - Q_5k_1, \quad P_2 = Q_2k_1 - h_2g_1 + x^2g_3, \quad P_1 = h_1g_1, \quad P_0 = x^2g_1,$$

$$z \equiv \tan(\omega - \delta) = \frac{x - x_2}{y + b + c - c(x_2/r)^2}. \quad (12)$$

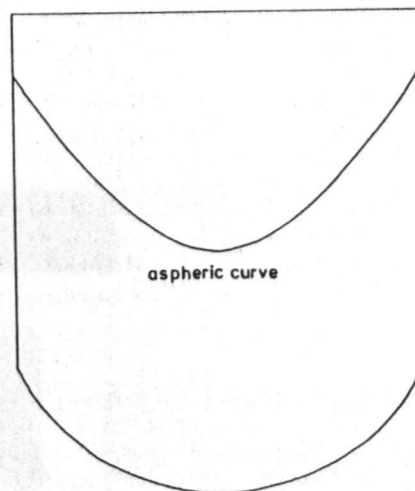


Fig. 2. Theoretically calculated aspheric curve for the corneal lens of *Notonecta glauca*.



Fig. 3. Interference micrograph of a section through a corneal facet of *Notonecta glauca* (courtesy of Rudolf Schwind).³ The bell shaped (dark) aspheric layer can be seen well.

Since the numerical values of the mentioned parameters of the corneal lens and crystalline cone of *Notonecta glauca* are known,^{2,3} n_1 (water) = 1.333 or n_1 (air) = 1, n_2 = 1.54, n_3 = 1.46, n_4 = 1.35, L = 75.88 μm , r = 19.41 μm , a = 22.94 μm , b = 10.59 μm , c = 12.35 μm , d = 0.88 μm . We computed Eqs. (10) and (11) and the result is shown in Fig. 2, while Fig. 3, an interference microphotograph of the corneal lens of *Notonecta glauca*,³ demonstrates that our optical model describes the structure of the lens of the backswimmer well. We hope that further development of our model will lead to the design of a photographic lens that performs equally well in water and air.

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