RESISTIVITY INCREASE DUE TO SIZE AND ROUGHNESS EFFECTS AND STABILIZATION OF IN SITU SUPERCONDUCTORS

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A simple geometrical model is given for the resistivity increase of thin conducting films. The results of our model agree well with the results of the non-geometrical Fuchs–Sondheimer theory. The model is improved for the films with roughness, when additional terms appear in the total resistivity expression. A quantitative analysis is presented for the thickness dependence of these additional terms under the condition of symmetrical deviations from the mean film thickness *d*. A calculation including the size effect is made for the normal resistivity of the in situ filamentary superconducting composite tapes. The size effect does not seem to play an important role in the stabilization of these discontinuous filamentary systems, not as it can play in continuous filamentary systems.

1. Theory and occurrence of the size effect

1.1 Size effect

If the thickness of a conducting film is commensurable with the mean free path of the conduction electrons in a bulk material, the electrical resistivity increases because of the diffusive scattering and the reflection of the electrons at the film surfaces. This resistivity increase is called size effect in this work.

The theory of the size effect is elaborated by Fuchs [1] for the free-electron model and a spherical Fermi surface. Price [2] improved this theory for ellipsoidal Fermi surfaces. Sondheimer [3] elaborated the Fuchs theory for the explanation of the galvanomagnetical effects. The expressions derived from the Fuchs–Sondheimer theory for the electrical conduction of thin conducting films can be used well in practice [4–8].

The size effect occurs practically for every very thin film because they have an insulating (oxide) layer or an adsorbed gas layer on their surface, so the scattering of the electrons at the film surfaces is partly or totally diffusive [5–8]. The influence of the adsorbed gas layer on resistivity of the thin metal films is examined by Finzel et al [9]. The size effect occurs also in every polycrystalline metal because the resistivity of the grain boundaries is larger than that of the crystallite. However, the resistivity increase is not significant, because the ratio of the resistivity of the crystallite and the grain boundary usually is near to unity [17–19].

The size effect appears in the multifilamentary superconducting composites, too, in normal state when the filaments are appropriately close to each other, and the normal resistivity of the filaments is much larger than the resistivity of the matrix [10–15]. A model is given by Cavalloni et al [13,14] to calculate the influence of the size effect on these multifilamentary systems.

In the granular (in situ) superconducting composites the size effect could play a greater role in the normal state than in the continuous filamentary systems because of the random distribution of the small grains or short filaments [16]. In such percolative systems the filaments can be very close to each other, so the interfilamentary resistivity of the very thin matrix layers can be hardly increased in normal state. The normal resistivity can nevertheless play an important role in the stabilization of the superconducting state of multifilamentary systems [16].

In this work we devise first a simple geometrical model to calculate the resistivity increase due to the size effect. We examine the conducting films with parallel surfaces which have insulating layers on their surfaces. We suppose that the electrons are scattered totally diffusively on the boundary surfaces of the film. We compare the results of our model to the result of the Fuchs–Sondheimer non-geometrical theory.

1.2. Size effect in a thin conducting film between insulating layers

In the free-electron model the resistivity is

$$\rho = m_0\overline{v}/(ne^2\lambda), \tag{1}$$

where m_0, e, n, $\overline{v}$ and λ are the mass, the charge, the volume concentration, the mean velocity and the bulk mean free path of the conduction electrons, respectively. The electron mean free path in the proximity of the surface of a finite conductor decreases because of the diffusive electron scattering on the scattering centres of the surface, and the resistivity of this part of the material increases.

Our geometrical model for the calculation of this resistivity increase is a conducting film between two parallel insulating layers with infinite resistivity. Because of this infinite resistivity the conduction electrons cannot be scattered beyond the boundaries of the film. Imagine a sphere with radius λ around the electrons, named

λ-sphere in this work. On average the electrons are scattered if they have reached this sphere surface. In a homogeneous, isotrope bulk material the electron mean free path is isotrope because of the infinite extension. However, in a very thin conducting film between the above mentioned insulating layers the boundary surfaces can cut a spherical calotte out of the λ-sphere. So the mean free path becomes direction dependent.

The average of this direction dependent mean free path can be calculated. Such a lambda sphere is cylindrically symmetrical to the axis T going through the examined electron and perpendicular to the insulating layers (Fig. 1), so it is enough to make an average for the main section.

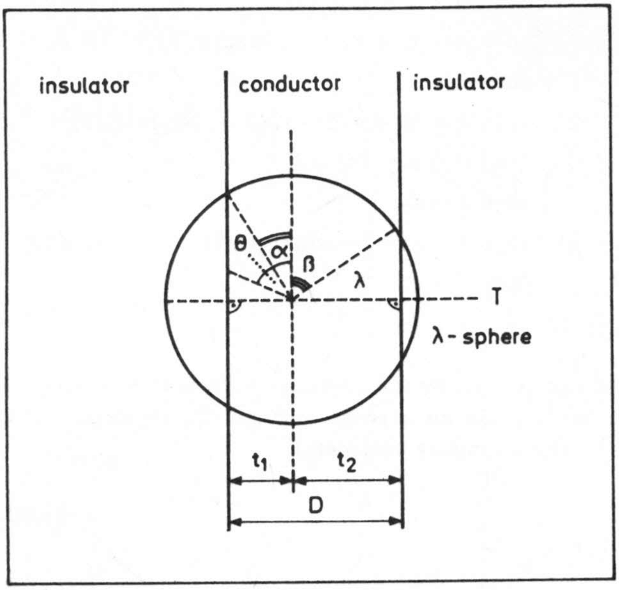

Fig. 1. The lambda-sphere in a thin conducting film between two parallel insulating layers

Consider an electron at a distance t_1 and t_2 from the film surfaces. In this case the average of the mean free path is

$$\overline{\lambda} = \left[\int_0^{\alpha} \lambda d\Theta + \int_0^{\beta} \lambda d\Theta + \int_{\alpha}^{\pi/2} (t_1/\sin\Theta) d\Theta + \int_{\beta}^{\pi/2} (t_2/\sin\Theta) d\Theta \right] \pi^{-1}, \tag{2}$$

taking into account of the notation of Fig. 1. Performing the integration we get from (2):

$$\overline{\lambda} = [\lambda(\alpha+\beta) - t_1 \ln \mathrm{tg}(\alpha/2) - (D - t_2) \ln \mathrm{tg}(\beta/2)]\, \pi^{-1}, \tag{3}$$

where D is the thickness of the film. Introduce the following dimensionless quantities

$$Q = D/(2\lambda), \qquad x = t_1/\lambda, \qquad \ell = \lambda/\overline{\lambda}\,. \tag{4}$$

In the case of very thin films both of the boundary surfaces could cut into the λ-sphere. In the case of thicker films such special arrangements exist when only one or none of the surfaces can cut into the λ-sphere. Taking into consideration the different arrangements, the average of ℓ depends on the quantities Q and x as follows

$$\begin{array}{ll}
1.\ \text{if } Q > 1 & \text{and } 0 \le x \le 1, \\
 & \text{then } \ell = [\alpha + \pi/2 - x \ln \mathrm{tg}(\alpha/2)]\pi^{-1} \equiv \ell_1 \\
1 < x < 2Q - 1, & \text{then } \ell = 1 \\
2Q - 1 \le x \le 2Q, & \text{then } \ell = [\beta + \pi/2 - (2Q - x) \ln \mathrm{tg}(\beta/2)]\pi^{-1} \equiv \ell_2 \\
2.\ \text{if } 1/2 \le Q \le 1 & \text{and } 0 \le x \le 2Q - 1, \\
 & \text{then } \ell = \ell_1 \\
2Q - 1 < x < 1, & \text{then } \ell = [\alpha + \beta - x \ln \mathrm{tg}(\alpha/2) - (2Q - x) \ln \mathrm{tg}(\beta/2)]\pi^{-1} \equiv \ell_3 \\
1 \le x \le 2Q, & \text{then } \ell = \ell_2 \\
3.\ \text{if } 0 \le Q \le 1/2, & \text{then } \ell = \ell_3\,.
\end{array} \tag{5}$$

If the conductor surrounded by the insulator with infinite resistivity has no parallel boundaries, we must make an average of λ for the residual λ-sphere without the part(s) cut off by the boundary surface(s).

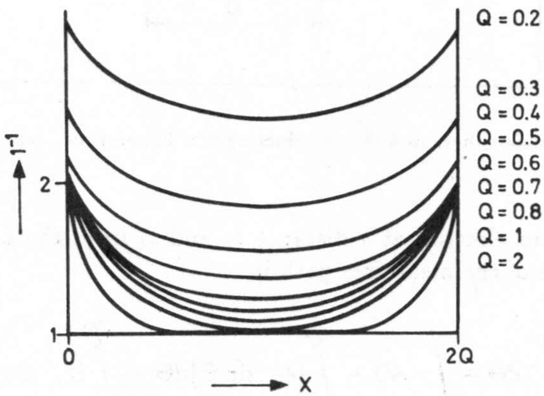

Fig. 2. Dependence of the dimensionless resistivity $\ell^{-1}(x, Q) = \lambda/\overline{\lambda}$ on $x = t_1/\lambda$ and $Q = D/(2\lambda)$

The dimensionless resistivity $\ell^{-1}(x, Q)$ is plotted in Fig. 2. As we can see at larger values of Q the resistivity increases only in the proximity of the surfaces.

The onset of the increase is at distance λ from the insulating layers. For distances larger than λ the resistivity is equal to the bulk one. For small Q the resistivity increase is extended over the whole film.

The average resistivity of the film is

$$\bar{\rho}^{-1} = Q^{-1} \int_0^Q [\rho(x)]^{-1} dx\,. \tag{6}$$

Using (1) and (6) we get

$$\bar{\rho}^{-1} = (\rho Q)^{-1} \int_0^Q \ell(x) dx, \tag{7}$$

where the functions $\rho(x)$, $\ell(x)$ are symmetrical. Using (4), (5), (7) we obtain the following expressions

1. $0 \le Q \le 1/2$

$$\bar{\rho}^{-1} = (\rho Q \pi)^{-1} \int_0^Q \Big\{ \arcsin x + \arcsin(2Q - x) -$$

$$- x \ln \operatorname{tg}\left(\frac{\arcsin x}{2}\right) - (2Q - x) \ln \operatorname{tg}\left[\frac{\arcsin(2Q - x)}{2}\right] \Big\} dx \equiv$$

$$\equiv (\rho Q \pi)^{-1} \int_0^Q F(x) dx\,,$$

2. $1/2 < Q < 1$

$$\bar{\rho}^{-1} = (\rho Q \pi)^{-1} \Big\{ \int_0^{2Q-1} \left[\pi/2 + \arcsin x - x \ln \operatorname{tg}\left(\frac{\arcsin x}{2}\right)\right] dx +$$

$$+ \int_{2Q-1}^{Q} F(x) dx \Big\} \equiv$$

$$\equiv (\rho Q \pi)^{-1} \Big\{ \int_0^{2Q-1} G(x) dx + \int_{2Q-1}^{Q} F(x) dx \Big\},$$

3. $Q \ge 1$

$$\bar{\rho}^{-1} = (\rho Q)^{-1} \Big[\pi^{-1} \int_0^1 G(x) dx + \int_1^Q dx \Big]. \tag{8}$$

Integrating (8), the average resistivities are

$$
\begin{aligned}
&1.\ 0 < Q < 1/2\\
&\quad \bar{\rho}^{-1} = (\rho Q \pi)^{-1}\Big\{3(1-4Q^2)^{1/2}/2 - (1-Q^2)^{1/2} - 1/2+\\
&\qquad + 2Q\arcsin(2Q) - 2Q^2 \ln\left[\frac{1-(1-4Q^2)^{1-2}}{2Q}\right]\Big\},\\
&2\text{–}3.\ 1/2 \le Q\\
&\quad \bar{\rho}/\rho = 1 + (2\pi Q - 1)^{-1}.
\end{aligned} \tag{9}
$$

On the basis of the Fuchs–Sondheimer non-geometrical theory for the resistivity ratio can be obtained [1–4]

$$
\begin{aligned}
\bar{\rho}/\rho &\approx 1 + 3/(16Q) = 1 + 0.1875/Q, && Q \gg 1,\\
\bar{\rho}/\rho &\approx -4/(3Q\ln Q) = -1.3330/(Q\ln Q), && 0 < Q \ll 1,
\end{aligned} \tag{10}
$$

when the scattering of the electrons is totally diffusive at the film surfaces.

From (9) we get

$$
\begin{aligned}
\bar{\rho}/\rho &\approx 1 + 1/(2\pi Q) = 1 + 0.1590/Q, && Q \gg 1,\\
\bar{\rho}/\rho &\approx -\pi/(2Q\ln Q) = -1.5710/(Q\ln Q), && 0 < Q \ll 1.
\end{aligned} \tag{11}
$$

It can be seen from (11) that the results of our simple geometrical model agree well with the results of the Fuchs–Sondheimer non-geometrical theory.

2. Influence of the size effect on rough films

2.1 Size effect in rough films

The influence of the surface roughness on the electrical properties of pure metal films is well known [15, 20–26]. The thickness dependence of the electrical resistivity of rough metal films is investigated thoroughly by Finzel and Wissmann [15,21]. The expression of the total resistivity of rough metal films consists of the contributions of the normal bulk resistivity, the well-known term of the scattering of conduction electrons on the grain boundaries and surfaces, and an additional term due to roughness, which varies proportionally to d^{-3} at symmetrical deviations of the thickness from the mean value d [21]. Finzel and Wissmann calculated the total resistivity of rough metal films in the case of $\lambda \ll B \ll d$, where B is the maximal thickness deviation from the mean value d [21]. Crittenden and Hoffmann introduced a thickness correction term into the total resistivity expression to describe the influence of the surface roughness. Namba [23] extended this theory assuming that the film surface can be characterized by a sinusoidal profile [22]. Hoffmann and Vancea gave a quantitative analysis of the roughness effect with roughness $B \ll \lambda \ll d$ [24,25,26]. The total resistivity of very thin films ($d \ll \lambda$) with surface roughness ($B \ll d$) is not investigated yet. In the case of $B \ll d \ll \lambda$ the size effect has an important role, too [2,3,10,13].

In this work we give a quantitative analysis for the thickness dependence of the electrical resistivity of very thin ($d \ll \lambda$) and rough ($B \ll d$) films considering the size and roughness effects. For the surface roughness we use the model of Finzel and Wissmann [21], assuming that the investigated very thin film consists of a monolayer of crystallites, and the boundaries of the cystallites are mainly align perpendicular to the film surface.

The average crystallite size and film thickness is d. The deviations of the thickness from this average value have a distribution function $f(y)$. This distribution has a cut-off at $y = B$ where B is the maximum value of these deviations. We assume a symmetrical distribution of the film thickness around the mean value d. Such a distribution has been confirmed experimentally by several authors [21,27,28].

2.2. Thickness dependent resistivity of thin rough films

The resistivity of a crystallite with size $d - y$ is [21]

$$\rho_c(y) = \rho_0 \left(1 + \frac{k}{d-y}\right) \quad , \quad k = z\sigma\lambda, \tag{12}$$

where z, σ, λ, d, y, ρ_0 are the number of scattering centres per unit surface, the scattering cross section, the bulk mean free path of the conduction electrons, the mean crystallite size, the deviation of the thickness from its mean value d, and the resistivity of very thin crystallites, respectively. By the model of Finzel and Wissmann [21] the total resistivity of the film is the average of the resistivity contributions of crystallites of different size

$$\rho_f = \frac{\int\limits_{-B}^{B} \rho_c(y) f(y) dy}{\int\limits_{-B}^{B} f(y) dy}, \tag{13}$$

where $f(y)$ is the distribution function of the grain size. In very thin films the resistivity ρ_0 increases because of the size effect. On the basis of the Fuchs–Sondheimer theory [2,3,7] the resistivity ρ_0 is in the case of $d \ll \lambda$

$$\rho_0 = \rho_b \left[1 + \frac{A\lambda}{(d-y)\ln\left(\frac{\lambda}{d-y}\right)}\right], \quad A = \frac{4(1-pq)}{3(1+p)(1+q)}, \tag{14}$$

where ρ_b is the resistivity of the bulk material with the same densities of lattice defects as it is in the film. The parameters p and q give the proportion of the conduction electrons scattered elastically on the two boundary surfaces of the grains. Introduce the notation

$$x = y/B, \tag{15}$$

and from the equations (12)–(15) we obtain

$$\rho_f = \rho_b \frac{\int\limits_{-1}^{1} \left(1 + \frac{k/d}{1-Bx/d}\right)\left[1 + \frac{A\lambda/d}{(1-Bx/d)\ln\left(\frac{\lambda/d}{1-Bx/d}\right)}\right] f(x)dx}{\int\limits_{-1}^{1} f(x)dx}. \tag{16}$$

The denominators in (16)

$$(1 - Bx/d)^{-1}, \qquad \left[(1 - Bx/d)\ln\left(\frac{\lambda/d}{1 - Bx/d}\right)\right]^{-1} \tag{17}$$

can be expanded in a power series of Bx/d because $Bx/d \ll 1$ is true in the case of $-1 \ll x \ll 1$ and $B/d \ll 1$. When the distribution function $f(y)$ is symmetrical, all the odd terms of the expansion vanish by the integration, so we can write

$$\begin{aligned}\rho_f &= \rho_b + \frac{k}{d}\rho_b + \frac{B^2G_2k}{d^3}\rho_b + \frac{A\lambda}{d\ln(\lambda/d)}\rho_b + \frac{Ak\lambda}{d^2\ln(\lambda/d)}\rho_b + \\ &+ \frac{B^2G_2A\lambda}{2d^3\ln(\lambda/d)}\left[2 - \frac{3}{\ln(\lambda/d)} + \frac{2}{\ln^2(\lambda/d)}\right]\rho_b + \dots = \\ &= \rho_b + \rho_{sc} + \rho_r^{(sc)} + \rho_{si} + \rho_{si}^{(sc)} + \rho_{si}^{(r)} + \dots ,\end{aligned} \tag{18}$$

where

$$G_n = \frac{\int\limits_{-1}^{1} x^n f(x)dx}{\int\limits_{-1}^{1} f(x)dx}, \qquad n = 2, 4, \dots . \tag{19}$$

In the case of $d \gg k$ the terms of higher order can be neglected compared with the first six terms in (18).

In the case of asymmetrical distribution function $f(y)$ further resistivity terms appear in the total resistivity, which can be calculated from (16)

$$\rho_1 = \frac{BkG_1}{d^2}\rho_b, \qquad \rho_2 = \frac{ABk\lambda G_1}{d^3\ln(\lambda/d)}\left[2 - \frac{1}{\ln(\lambda/d)}\right]\rho_b,$$

$$\rho_3 = \frac{AB\lambda G_1}{d^2\ln(\lambda/d)}\left[1 - \frac{1}{\ln(\lambda/d)}\right]\rho_b. \tag{20}$$

Expressions (18) and (20) give the mean thickness dependence of the additional resistivity terms due to the roughness and size effect considering a symmetrical and asymmetrical distribution function $f(y)$, respectively.

3. Size effect on in situ composites

3.1 Size effect in the in situ superconducting composites

In an in situ superconductor the discontinuous filaments can be very close to each other. Between the filaments a normal matrix film can be formed. In the normal state of a superconducting wire the normal resistivity will be determined by the matrix resistivity, because the normal resistivity of superconducting filaments is much higher. The appropriate normal resistivity of the multifilamentary superconductors is a very important factor in the stabilization of superconducting systems. In this work we investigate the role of the size effect in the normal resistivity of the in situ filamentary superconductors.

The resistivity of the multifilamentary superconducting wires or tapes, in the normal state is one of the most important parameters for the design and the optimization of a superconducting system. When the multifilamentary conductor goes to normal state because of some irregularities the current will be carried by the whole volume of the conductor. To decrease the current consumption of the superconducting filaments the longitudinal resistivity of the matrix has to be much smaller. In this case the important part of the current can be carried by the non-superconducting matrix. In AC electrical or magnetic fields there is an essential contribution to the energy losses from the arising eddy currents. These currents flow in transverse direction and if we want to decrease the losses the transverse component of the normal matrix resistivity has to be large to eliminate the eddy currents. So the longitudinal resistivity $\rho_{||}$ of the multifilamentary superconducting composites in the normal state determines the current flow dissipation, and the transverse resistivity $\rho_{\perp}$ is an important parameter of the composite superconductor in relation to the AC losses. At present the wires and tapes consisting of a few thousand of different superconducting thin filaments in a normal metal matrix with small residual resistivity are most frequently used.

Several experimental and theoretical works on the superconducting wires and tapes have been accumulated [10–16,29–37] till now, but the importance of size effect on the normal resistivity of such superconducting systems has been fully recognized only recently. Drobin et al developed a method to measure directly the transverse ($\rho_{\perp}$) and the longitudinal ($\rho_{||}$) resistivities of a normal matrix in commercial continuous multifilamentary superconducting wires and tapes. They investigated also the size and proximity effects in these systems [10,15]. There are several other methods for the indirect determination of $\rho_{\perp}$ and $\rho_{||}$ [29,30]. Taking into account the size effect, Cavalloni et al performed numerical calculations of $\rho_{\perp}$ and $\rho_{||}$ of the multifilamentary superconducting wires in the normal state [13,14]. Other authors investigated also the normal resistivity of these wires and tapes considering the size effect [11,12,13].

The size effect can play a more important role in the interfilamentary resistivity of the granular or filamentary (called in situ) superconducting composites, because the superconducting filaments can be very close to each other in these systems. Several theoretical and experimental works have been made on the normal resistivity of the in situ filamentary superconducting composites [16,32–37], but the size effect on the electron transport in the normal state is not investigated in these systems yet.

As we mentioned the transverse resistivity of the matrix has to be as large as possible in order to suppress the eddy currents responsible for a sizable fraction of the AC losses. On the other hand, the longitudinal resistivity of the matrix must be as small as possible, to minimize the power dissipation and the concomitant temperature rising if some regions of the superconductor suddenly go to normal state, when practically all the current will be carried by the matrix [13].

In this work we show for the in situ filamentary superconducting composite tapes that the transverse resistivity of the matrix will not be large in spite of the fact that the interfilamentary resistivity can be very large because of the size effect. The longitudinal resistivity of the matrix can be suitably small in spite of the resistivity increase between the filaments.

3.2. Calculation of the average resistivity of the in situ composite considering the size effect

We investigate such filamentary superconducting composites in which the resistivity of the superconducting filaments in the normal state is much larger than that of the matrix, and therefore it can be taken as infinite. In the in situ filamentary superconducting composite tapes the long flat filaments are randomly distributed in a normal metal matrix. We consider a diffusive electron scattering at the matrix–superconductor interface in the normal state of the superconductor and an isotropic bulk electron mean free path. Under these circumstances the theory of Fuchs–Sondheimer can be used to describe the resistivity increase between the overlapping filaments. On the basis of this theory the interfilamentary resistivity is

$$\rho_i = \rho_b \left[1 + \frac{8\lambda}{3D \ln\left(\frac{2\lambda}{D}\right)} \right], \qquad D \ll \lambda, \tag{21}$$

where ρ_b, λ and D are the resistivity of the bulk matrix, the isotropic bulk electron mean free path and the distance between the overlapping filaments, respectively. If the tape is very elongated and/or flattened the condition $D \ll \lambda$ can be realized.

Using Carr's model [33–35], we consider an in situ filamentary superconducting composite as a continuum with anisotropic properties averaged over the volume containing a large number of superconducting filaments. In this model only the resistivity of a uniform cell must be investigated, and the composite consists of such uniform cells. The average resistivity of the composite is equal to the resistivity of one cell.

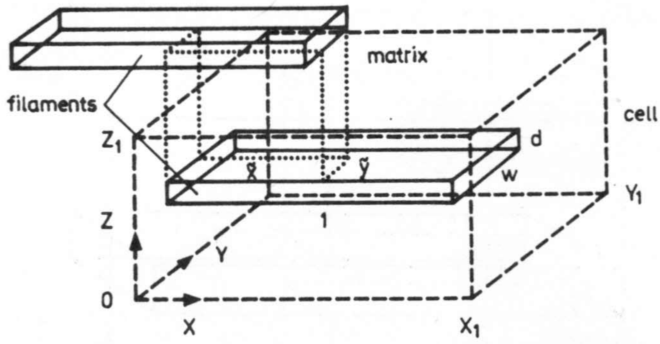

Fig. 3. Two neighbouring superconducting filaments overlapped, the interfilamentary region of the matrix and a uniform cell of the in situ composite

A uniform cell of the in situ filamentary superconducting composite tape can be seen in Fig. 3, where x_1, y_1, z_1, ℓ, w, d, $\tilde{x}\cdot\tilde{y}$ are the side measurements of the uniform cell and the filament in the direction x, y, z and the area of the overlapping interfilamentary region, respectively. The filament is placed in the uniform cell coaxially. ρ_b, ρ_f and ρ_i are the resistivities of the matrix, the filament and the overlapping interfilamentary region, respectively. The resistivity ρ_f will be taken as infinite, the resistivity ρ_i is increased by the size effect as it can be seen in Eq. (21). The direction of the elongation of the composite is the direction of the axis x; and the composite is flattened parallel to the surface xy. We suppose $\ell \gg w \gg d$, so a significant size effect occurs only between the overlapping surfaces $\tilde{x}\tilde{y}$ of the neighbouring filaments. The distribution of the resistivity in a cell can be seen in Fig. 4. Assuming parallel current lines to surfaces, the average resistivity of a uniform cell in the direction x, for example is

$$\frac{1}{R_x} = \int_0^{z_1}\int_0^{y_1} \frac{dydz}{\int_0^{x1} \rho(\underline{r})dx} . \tag{22}$$

On the basis of Fig. 4 performing the integrations, we obtain

$$\frac{1}{R_x} = \frac{z_1(y_1-\tilde{y})+(\tilde{y}-w)d}{\rho_b x_1} + \frac{wd}{\rho_b(x_1-\ell)+\rho_f\ell} + \frac{(z_1-d)\tilde{y}}{\rho_i\tilde{x}+\rho_b(x_1-\tilde{x})}, \tag{23}$$

$$\frac{1}{R_y} = \frac{z_1(x_1-\tilde{x})+(\tilde{x}-\ell)d}{\rho_b y_1} + \frac{\ell d}{\rho_b(y_1-w)+\rho_f w} + \frac{(z_1-d)\tilde{x}}{\rho_i\tilde{y}+\rho_b(y_1-\tilde{y})}, \tag{24}$$

$$\frac{1}{R_z} = \frac{x_1y_1-w\ell}{\rho_b z_1} + \frac{\tilde{x}\tilde{y}}{\rho_i(z_1-d)+\rho_f d} + \frac{w\ell-\tilde{x}\tilde{y}}{\rho_b(z_1-d)+\rho_f d}. \tag{25}$$

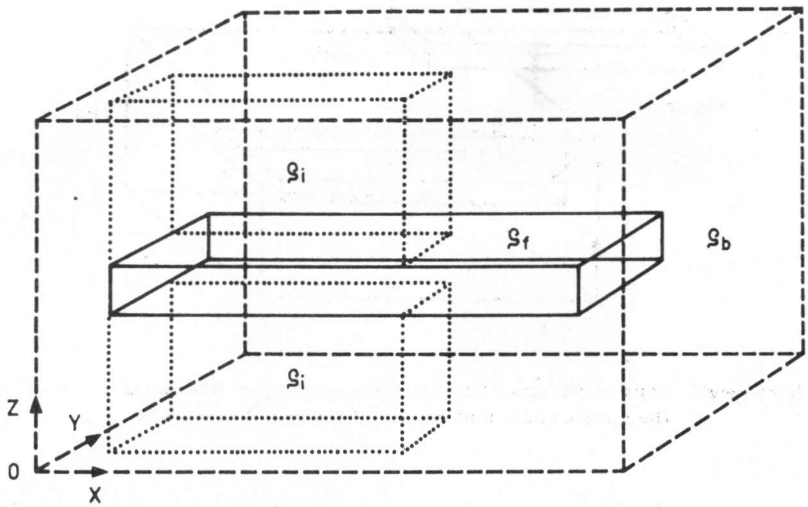

Fig. 4. The distribution of the normal resistivity $\rho(x,y,z)$ in a cell of the composite. The origin of the system of co-ordinates is the point 0

An in situ filamentary superconducting tape is elongated and flattened from an initial granular superconducting composite. For the elongation and the flattening the following terms can be written

$$x_1 y_1 z_1 = x_0^3 = \ell w d/c = \ell_0^3/c, \tag{26}$$

$$E = \ell/\ell_0 = x_1/x_0, \quad y_1/x_0 = w/\ell_0, \quad z_1/x_0 = d/\ell_0, \tag{27}$$

$$k = w/d, \tag{28}$$

where x_0, ℓ_0, c, E, k are the initial length of the composite, the average grain size, the superconducting volume ratio, the degree of elongation and flattening, respectively. From (26), (27), (28) we obtain

$$\ell = \ell_0 E, \qquad w = \ell_0 (k/E)^{1/2}, \qquad d = \ell_0 (kE)^{-1/2},$$

$$x_1 = c^{-1/3}\ell_0 E, \quad y_1 = c^{-1/3}\ell_0 (k/E)^{1/2}, \quad z_1 = c^{-1/3}\ell_0 (kE)^{-1/2}. \tag{29}$$

Using (29), the average resistivity of a uniform cell, in the directions x, y, z is

$$\begin{aligned} \rho_x &= R_x y_1 z_1/x_1 = R_x \ell_0/(E^2 c^{1/3}), \\ \rho_y &= R_y x_1 z_1/y_1 = R_y \ell_0 E/(k c^{1/3}), \\ \rho_z &= R_z x_1 y_1/z_1 = R_z \ell_0 E k c^{-1/3}. \end{aligned} \tag{30}$$

The filaments have a random distribution in the matrix. This can be taken into account in Carr's model, so an average overlapping area is supposed with the following average side measurements

$$\bar{\bar{x}} = n_x \ell \quad \text{and} \quad \bar{\bar{y}} = n_y w \ , \ 0 < n_x n_y < 1. \tag{31}$$

The value of n_x and n_y increases with the increase of the superconducting volume concentration c. Using (23), (24), (25), (29), (30), (31) and substituting $\rho_f = \infty$ into (23), (24), (25) we obtain the direction dependent average resistivity of the in situ filamentary superconducting tape

$$\rho_x = \rho_b \left\{ (1 - c^{1/3})[1 - c^{1/3}(n_y - 1)] + \frac{n_y(1 - c^{1/3})}{\frac{n_x \rho_i}{\rho_b} + c^{-1/3} - n_x} \right\}^{-1}, \tag{32}$$

$$\rho_y = \rho_b \left\{ (1 - c^{1/3})[1 - c^{1/3}(n_x - 1)] + \frac{n_x(1 - c^{1/3})}{\frac{n_y \rho_i}{\rho_b} + c^{-1/3} - n_y} \right\}^{-1}, \tag{33}$$

$$\rho_z = \rho_b / (1 - c^{2/3}). \tag{34}$$

Since the composite is elongated in the direction x ($\ell \gg w \gg d$), the longitudinal resistivity is described by the expression of ρ_x, and the transverse resistivities are described by the expressions of ρ_y and ρ_z. It can be seen that the size effect is important only in the longitudinal resistivity and the transverse resistivity of direction y.

In Carr's model the average distance between the overlapping filaments is

$$D = z_1 - d = \ell_0 (kE)^{-1/2} (c^{-1/3} - 1). \tag{35}$$

Size effect occurs if $D \ll \lambda$. From (21) and (33) it can be seen that the condition $\rho_i/\rho_b \gg 1$ is confirmed. At a given ℓ_0 and c D can be small in the case of $kE \gg 1$. So the size effect can increase the interfilamentary resistivity for large elongation ($E \gg 1$) and/or for large flattening ($k \gg 1$) of a composite. In the case of large ρ_i/ρ_b the following expressions can be given for the average longitudinal resistivity and the transverse resistivity of direction y

$$\rho_x = \rho_b \left\{ (1 - c^{1/3})[c^{1/3}(1 - n_y) + 1] \right\}^{-1}, \tag{36}$$

$$\rho_y = \rho_b \left\{ (1 - c^{1/3})[c^{1/3}(1 - n_x) + 1] \right\}^{-1}. \tag{37}$$

Taking into account the relation $n_x \approx n_y$, because of the random distribution of the initial grains in the matrix, ρ_x and ρ_y are described by the same expression. So the requirement for ρ_y to be large while ρ_x is small, cannot be realized. When ρ_x is small then ρ_y is small too, and inversely.

From (34), (36) and (37) it can be seen that the average resistivity of a composite depends only on the superconducting volume ratio, so the size effect does not have a significant role in it. The size effect can make change the interfilamentary resistivity only.

4. Summary

The size effect occurs in many important systems: in very thin conducting films, in polycrystalline metals, in continuous filamentary and granular superconducting composites. This effect increases the resistivity more or less significantly in these systems. In this work we investigated the resistivity increase due to the size and roughness effects in different conducting systems.

We considered a thin plane-parallel conducting film with insulating layers. The resistivity increase in it is described well by our simple geometrical model and our results agree well with the results of the Fuchs–Sondheimer non-geometrical theory.

We investigated a thin rough conducting film. The assumption of our calculation was that the thickness deviations from the mean value d are symmetrical, the maximal deviation is much smaller than d, and d is much smaller than the bulk mean free path of the conduction electrons. Under these circumstances the total resistivity is given by six resistivity terms. The first three terms are well known: the normal bulk resistivity (ρ_b), the contribution of the scattering of the conduction electrons on the grain boundaries and surfaces (ρ_{sc}), and the term of the scattering on the roughness $(\rho_r^{(sc)})$. Our new additional resistivity terms belong to the size effect $(d \ll \lambda)$. The first additional term ρ_{si} is due to the size effect only, and varies as $[d \ln(\lambda/d)]^{-1}$. The second one $\rho_{si}^{(sc)}$ describes the combination of the size effect and the scattering of the electrons on the grain boundaries and surfaces, which varies as $[d^2 \ln(\lambda/d)]^{-1}$. The third additional term is due to the combination of the size and roughness effects, and it varies as $d^{-3}F[\ln(\lambda/d)]$. At very thin films $(d \ll \lambda)$ the additional three terms have primary relevance in the total resistivity. The parameters b, k, B, G_2, A and λ in the expression of the total resistivity can be evaluated from experimental data. When the thickness deviations from the mean value d are asymmetrical, some further terms appear in the total resistivity expression. These terms are due to the combination of the roughness effect and the scattering (on the grain boundaries and surfaces) (ρ_1), the size and roughness effect plus the scattering (ρ_2), the size and roughness effect (ρ_3).

Finally, we investigated the role of the size effect in the in situ superconductors. The interfilamentary normal resistivity can be very large in the in situ superconducting tapes because of the size effect if the elongation and the flattening of the tape are large. On the other hand the average normal transverse resistivity cannot be large in spite of the size effect, so the eddy currents, which are responsible for a sizable fraction of the AC losses cannot be suppressed. The average normal longitudinal resistivity remains small for a small value of the superconducting volume ratio in spite of the size effect, so all the current can be carried with small power dissipation by the matrix if some regions of the superconductor go to normal state. Thus, the size effect does not play an important role in the average normal resistivity of the in situ superconducting tapes, therefore the size effect cannot be used for the purpose of the stabilization as it is used in the case of the continuous filamentary superconductors. This conclusion can be explained by the difference of the structure of the in situ and the multifilamentary superconducting tapes. In the

in situ superconducting tapes there is a random distribution of the finite filament fragments and in this case the appropriate current distribution does not take into account the parts of the conductor with locally enlarged interfilamentary resistivity. This can be an interpretation for the insensitivity of the size effect of the in situ superconducting tapes.

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